

## Monopoly in a competitive setting

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(Recibido: enero 2018/Aceptado: junio 2018 )

### Abstract

A production market with given preferences, technology and competition technology is vulnerable if it admits both perfect competition and monopoly or oligopoly. Under decreasing returns, sunk costs combined with a potential for monopoly profits provide a sufficient basis for vulnerability. A large agent can establish monopoly by installing enough productive capacity. The monopolist deters entry by threatening to oversupply the market. The threat is credible if the future discount rate is low enough and if enough small players enter the market in the absence of punishment. Financial institutions can capture vulnerable markets for profit, reducing competition, efficiency and equity.

*Keywords:* monopoly, competitive markets, competition technology.

*JEL classification:* L12.

## El monopolio en un entorno competitivo

### Resumen

Un mercado de producción con preferencias dadas, tecnología y tecnología de competencia es vulnerable si admite tanto la competencia perfecta como el monopolio u oligopolio. Con los rendimientos decrecientes, los costos irre recuperables combinados con un potencial de ganancias monopolísticas

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proporcionan una base suficiente para la vulnerabilidad. Un gran agente puede establecer un monopolio instalando suficiente capacidad productiva. El monopolista disuade la entrada al amenazar con abastecer excesivamente al mercado. La amenaza es creíble si la tasa de descuento futura es lo suficientemente baja y si suficientes jugadores pequeños ingresan al mercado en ausencia del castigo. Las instituciones financieras pueden capturar mercados vulnerables con fines de lucro, reduciendo la competencia, eficiencia y equidad.

*Palabras claves:* monopolio, mercados competitivos, tecnología de competencia  
*Clasificación JEL:* L12.

## 1. Introduction

This paper asks the question: in an otherwise competitive context, can the size of the participants determine the structure of the market? More specifically, can a large participant take over a competitive market, impose higher prices and obtain a profit? The answer is clearly not if instantaneous production and sale are possible without sunk costs. Thus I study the case when production requires a sunk cost that serves to signal a participants' intent to produce and makes her vulnerable to a loss. A large producer can then deter other participants by threatening to oversupply the market. When there is a potential for monopoly profits and the threat is credible, a large producer can make a profit over and above her expenditures on deterrence.

The paper demonstrates the existence of multiple types of market equilibria for a wide class of productive contexts with decreasing returns to scale, in which producers facing many buyers compete by selling their product. One type of equilibrium consists of many small owners. In the presence of a large enough agent, though, a monopolic equilibrium also exists. The equilibrium with many small owners is usually called perfect competition. However, the difference between the two types of equilibria resides in the distribution of ownership and not in the manner or technology of *competition*. Thus I call the equilibrium with many small owners *demopoly*.<sup>1</sup>

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<sup>1</sup> Monopoly means "one seller," oligopoly "few sellers." Many sellers would be polipoly, which does not sound very good. Thus I use demopoly instead, meaning "the people sell," on the suggestion of Sonia Di Giannatale Menegalli. I also thank Luciana Moscoso Boedo for useful comments on the model during the writing of this article.

Under monopoly, an additional *strategy* appears for competing by selling a product than under demopoly. It is possible for a big producer to threaten other producers to oversupply the market so as to cause them a loss. I define the concept of *deterrent monopoly*, when an incumbent incurs a flow of expenditure to deter entry by other participants and therefore makes a profit. I define also the concept of *vulnerable market*, a market which has a demopolic (or competitive) equilibrium but that can also be captured by a monopoly or oligopoly. In the current paper, we show that the following four assumptions (in addition to decreasing returns) are sufficient for a market to be vulnerable to capture by a deterrent monopoly.

- 1) *Production involves sunk costs.* An example is capital investment. Sunk costs make participants vulnerable to a threat.
- 2) *Aggregate income from sales rises when supply is decreased.* This is necessary for there to be an incentive to monopolize a market.
- 3) *Even a small deviation by a large agent from her costly punishment threat will cause a discrete measure of small agents to disregard the threat in the future.* This is a technical condition for a credible threat.
- 4) *Discount of the future is low enough that maintaining future profits justifies the cost of punishment.* This is a necessary condition for a credible threat.

Under these four conditions, demopoly will be the only market equilibrium when all agents are small enough. However, if there is a large enough agent, multiple equilibria will arise, amongst them deterrent monopoly. Equilibrium choice will depend on power play transcending demand and supply. For simplicity, I leave the case of *deterrent oligopoly* for future work.

Note that if an agent has enough credit, she will command enough resources to establish a deterrent monopoly. It follows that for vulnerable markets, perfect credit destabilizes demopoly, generating multiple equilibria in price and quantity, and making deterrent monopoly possible.

The model I present is related to several strands of literature. One strand, with a long history, studies the interaction between small and large players in a context of general equilibrium. Using the tools of cooperative game theory, Shi-tovitz (1973) follows Aumann (1964), as I do here, in the simultaneous modelling of small and large agents. The idea is to use a continuum of infinitesimal participants to represent small players (the ocean) and a finite set of participants with finite measures (atoms) to

represent large players. Concentrating on the core of the economy, which is the set of allocations that no coalition can improve upon by using only their own resources, Shitovitz obtained counterintuitive results in which oligopolistic outcomes are equivalent to competitive outcomes under quite wide assumptions. Okuno, Postlewaite and Roberts (1980) criticize this solution concept, obtaining quite different results for the Nash equilibrium in a noncooperative model of exchange. In their two-commodity exchange economy, large agents or syndicated groups restrict supply to obtain higher welfare in a sub-optimal equilibrium. Moreover, this approach also obviates the need to assume some agents are strategic while others are not. A related line of research, originated by Gabszewicz and Vial (1972), studies oligopoly ‘a la Cournot-Walras,’ with “few” oligopolists and “many” consumers. The original approach gives rise to a series of theoretical problems that have been successively overcome. Codognato and Gabszewicz (1991) define a Cournot-Walras equilibrium concept that does not depend on price normalization. Codognato (1995) and Codognato and Ghosal (2000) define instead a Cournot-Nash equilibrium concept that eliminates the asymmetry between strategic and non-strategic agents. Busetto, Codognato and Ghosal (2008) eliminate inconsistencies between these different types of models by harmonizing the number of stages and structure of play in the various settings. They then show that Cournot-Walras equilibria are equivalent to pseudo-Markov subgame perfect Nash equilibria, for which small players take only each commodity’s aggregate supply into account. Busetto, Codognato and Ghosal (2011) show the existence of a pure strategy Cournot-Nash equilibrium for a model of noncooperative exchange including large and small traders allowed to buy and sell all commodities.

The current paper also uses the subgame perfect Nash equilibria, but introduces threats in players’ strategies, not considered in the work we have mentioned. To do this it needs to introduce two additional elements, sunk costs and infinite repetition of the typical production round, so as to model the credibility of the threats. I therefore work with the simplest possible model. To be as realistic as possible, I keep to partial equilibrium in a single production market, rather than considering an exchange economy. As mentioned above, I also only consider the case of monopoly, leaving oligopoly for future work.

A second related strand of literature is predatory pricing, which in our model is the basis for the existence of deterrent monopoly. The predation

literature has a long history. A series of works support the theoretical possibility of predation, while a series of critics doubt the practical relevance of these theories. McGee (1958) argues that purchase of rival firms is cheaper and more reliable than predatory pricing. Discussing predation and noting McGee (1958), Yamey (1972) concludes that predatory pricing should be given a place in the analysis of barriers to entry. Persson (2004) gives arguments in a multifirm context as to why predation might be cheaper than mergers. The present day theory of predation is based on game theory, usually set in the context of imperfect competition (unlike our own context of small, competitive actors), and discusses the interaction of a series of market imperfections. For example Fudenberg & Tirole (1986) model a two-firm context in which predatory pricing has the function of jamming information for an entrant. Harrington (1989) proves that collusion and predation can exist in an oligopolistic setting. Bolton & Scharfstein (1990) show that predation can be motivated by financial contracting that threatens a firm with bankruptcy if it fails to meet payments. Milgrom & Roberts (1997) show that predation can be rational for a monopolist deterring several potential entrants in a reputation setting. Roth (1996) shows that predatory pricing is rationalizable (a generalization of Nash equilibria based on modelling reasonable beliefs). It is interesting to note that predatory pricing is robustly reported in experimental economics (Gomez & Goeree, 2008).

Our theory of vulnerable markets in a sense falls within this tradition. The theoretical possibility of predatory pricing supports the theoretical existence of vulnerable markets. Cheaper ways of deterring competition may be available in practice, but involve additional market imperfections. Common examples could be local or targeted predatory pricing against specific entrants, small economies of scale in production or innovation, mergers, hostile takeovers, even physical threats and legal harassment. These would provide other, perhaps more realistic ways, in which vulnerable markets are captured by monopolies or oligopolies. On the other hand the theoretical argument is clearer without the additional imperfections.

Indeed, the general structure of the argument for the existence of vulnerable markets is the following. A given good  $X$  is produced under conditions  $C$ . Suppose that under these conditions, as well as decreasing returns to scale, demopoly is possible and stable. All producers face the same risks and perils  $C$ . Nevertheless, it might still be possible for one or several large agents to establish a monopoly or oligopoly for the production of  $X$ , taking

advantage of conditions C to device strategies for deterring the participation of small agents. Thus, conditions C are consistent with demopoly but can also serve to support monopoly or oligopoly. In the present case conditions C are the four conditions listed above, but other settings are possible.

While in this paper I examine only the case of monopoly, the literature on oligopolistic collusion supports the idea that markets can be vulnerable to capture by an oligopoly. Abreu (1986), for example, shows that carrot and stick strategies can sustain oligopolistic collusion (optimally amongst symmetric strategies), which could be used to take over a vulnerable market. Collusion also exists under imperfect monitoring (Abreu, Pearce, and Stacchetti, 1986; Green and Porter, 1984). A series of other phenomena could also be involved, such as market share or price wars (Levenstein, 1993, Pot *et al.*, 2010). Osterdal (2003) shows that the stability of carrot and stick collusion strategies (without the restriction to symmetry) improves with the degree of product differentiation. Thus a set of big players could collude to subdivide a set of product markets, allotting to each of themselves a subset of products on which to produce as a monopolist. It follows that in a general equilibrium setting a set of markets for which stable demopolistic production is possible could be vulnerable to capture by monopolistic competition. Again, large financial institutions could provide the facilitating mechanisms for such a transformation from perfect to monopolistic competition, by providing the tools for a change in the ownership structure.

Finally, a discussion giving vulnerable markets an economic and historical setting is in order. First, market concentration has been the norm rather than the exception for US production during the 20th century. From 1935 to 1992, the average production of the four largest firms in 459 industries was 38.4% of all shipments. Similarly, from 1992 to 2002, the 200 largest manufacturing companies accounted for 40% of manufacturing value added.<sup>2</sup> Consistently with market power, Hall (1988) shows in a study of US industry that marginal cost is often well below price. Industrial concentration has also been a salient feature of globalization. In 2007, 89.3% of global FDI inflows consisted of mergers and acquisitions (UNCTAD, 2008). By 2008, the world's top 100 non-financial transnational corporations produced 14.1% of global

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<sup>2</sup> Data from U.S. Census Bureau-Economic Census. 1992. "Concentration Ratios for the U.S." <http://www.census.gov/epcd/www/concentration92-47.xls>.

output (*ibid*). There is therefore a risk that global concentration could continue to rise towards US levels. Concentration has also risen tremendously in agriculture (Hendrickson & Heffernan 2007) and finance (D'Arista, 2009).

However, industrial concentration only began in the US in the late 19th Century. The Sherman Antitrust Act, meant to prevent the destruction of competition through the formation of cartels and monopolies, itself dates to 1890. This was the time when a wave of mergers radically transformed the banking sectors of Boston and Providence (Lamoreaux, 1991). In addition, the first wave of mergers and acquisitions recognized by economic historians for the US took place from 1893 to 1904.<sup>3</sup> This period saw the birth of the main steel, telephone, oil, mining, railroad and other giants of the basic manufacturing and transportation industries. The major automobile manufacturers emerged during the second wave, from 1919 to 1929, featuring vertical integration that in the case of Ford reached all the way to the iron and coal mines. In the third period (1955 to 1969-73) the conglomerate concept took hold of American management. The fourth wave, already during the recent period of globalization, was the merger or takeover wave of the 1980s, which inaugurated the era of hostile takeover bids by major investment banks. It featured a new set of financial mechanisms such as junk bond financing and leveraged buyouts. The fifth wave (1993 to 2000) was the era of the mega-deal. From a modest \$342 billion in 1992, worldwide merger volume reached \$3.3 trillion in 2000. This wave ended with the NASDAQ collapse, but by 2006 a sixth merger wave was in full swing. By mid-2008, just before the financial crisis, five banks were emitting 97 percent of all derivative assets.

Vulnerable markets are markets for which there is an incentive to change the ownership structure from demopoly to monopoly or oligopoly. The history of merger waves, which began when sufficiently large economic actors appeared, is consistent with the theory of vulnerable markets proposed here, as well as with the role that financial markets can play in transforming competitive into concentrated production ownership structures.

In what follows I present the deterrent monopoly model and conclude.

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<sup>3</sup> This paragraph's summary of merger waves is based on Lipton (2006).



## 2. The model

### 2.1. Production

In a partial equilibrium model, consider a single market for a given good  $X$ , to be sold at price  $P$ . Suppose that production of  $X$  occurs on a continuum of plants  $i \in [0, N]$ , where  $N$  will be determined endogenously. In each plant, production of a quantity  $q$  of good  $X$  costs  $c(q)$ , where  $c'' > 0$ , and for simplicity  $c' > 0$  for all quantities (alternatively  $c'$  could begin negative and end positive). Let the inverse demand for good  $X$  is given by

$$P = P(Q), P'(Q) < 0, \quad (1)$$

where  $Q$  is the total quantity produced.

The optimal arrangement for any level of aggregate production  $Q$  consists of operating each plant at its optimal capacity  $q^*$ , and choosing the number of plants  $N$  so that  $Q = Nq^*$ . This is because the first order condition for the minimization problem

$$\begin{aligned} \min Nc(q), \\ \text{s.t. } Q = Nq \end{aligned} \quad (2)$$

after substituting for  $N$ , is

$$0 = \frac{d}{dq} \left[ \frac{Q}{q} c(q) \right] = \left[ -\frac{Q}{q^2} c(q) + \frac{Q}{q} c'(q) \right] \Leftrightarrow \frac{c(q)}{q} = c'(q) \quad (3)$$

Observe that since  $Q$  is fixed, the minimization problem is equivalent to minimizing average cost  $c(q)/q$ . The result is the well-known condition that average cost equals marginal cost, defining an optimal plant production level  $q^*$  where the elasticity of cost equals one,  $q^* c'(q^*)/c(q^*) = 1$ . Suppose  $q^*$  is unique. The second order sufficient condition for the minimum is satisfied,

$$\frac{d^2}{dq^2} \left[ \frac{Q}{q} c(q) \right] \Big|_{q^*} = 2\frac{Q}{q^3} c(q) - \frac{2Q}{q^2} c'(q) + \frac{Q}{q^*} c''(q^*) > 0 \quad (4)$$

Let the optimal average cost be

$$\gamma = \frac{c(q^*)}{q^*} \quad (5)$$



### 3. Competitive production

Consider for simplicity the case of competitive production in which each plant is run by a distinct owner  $i \in [0, N]$ , so each firm corresponds to a single plant. Entry consists of establishing a new plant. Each firm maximizes profits

$$\pi c(q) = Pq - c(q) \quad (6)$$

The first order condition for profit maximization of the price-taking firms is

$$0 = \pi c'(q) = P - c'(q) \quad (7)$$

that is, marginal cost  $c'(q)$  equals marginal revenue  $P$ . Now firm entry reduces profits to zero defining a total number of firms  $N$  according to

$$0 = \pi_c = P(Nq)q - c(q) \quad (8)$$

Combining this with equation (3) yields

$$c \frac{c(q)}{q} = P(Nq) = c'(q) \quad (9)$$

Hence  $q = q^*$ , which means competitive production is efficient, and the equilibrium number of firms  $N$  under perfect competition is:

$$N_c^* = \frac{1}{q^*} P^{-1}(\gamma) \quad (10)$$

#### 3.3. Monopolistic production

Consider now a single owner who establishes a continuum of plants  $i \in [0, N]$  and prices the good as a monopoly. As noted above, minimizing production costs implies each plant operates at the optimum level of production  $q^*$ . Hence the monopolist chooses the total production quantity  $Q = Nq^*$  by choosing the number of plants  $N$  so as to maximize profits. Total cost can be written

$$C(Q) = N_c(q^*) = Q\gamma \quad (11)$$

and marginal cost is  $c'(Q) = \gamma = \frac{c(q^*)}{q^*}$ . Total revenue can be written

$$TR(Q) = PQ = P(Q)Q \quad (12)$$

Hence the monopolist maximizes

$$\max_Q \pi_M(Q) = TR(Q) - Q\gamma \quad (13)$$

The first order condition occurs where marginal revenue equals marginal cost

$$TR'(Q) = \gamma = C'(Q) \quad (14)$$

Hence the quantity  $Q_M^*$  of production under monopoly is defined by

$$P(Q_M^*) + P'(Q_M^*)Q_M^* = \frac{c(q^*)}{q^*} \quad (15)$$

Let us compare competitive and monopolistic production. The equations for quantities produced are:

$$P(Q_c^*) = \gamma \quad (16)$$

$$P(Q_M^*) + P'(Q_M^*)Q_M^* = \gamma \quad (17)$$

Since  $P'(Q_M^*) < 0$ , it follows  $P_c^* \equiv P(Q_c^*) < P(Q_M^*) \equiv P_M^*$ . Hence  $Q_M^* < Q_c^*$ ; monopolistic price is higher and production lower than in the competitive case.

For simplicity in what follows we now define units so that the optimal production at each plant is  $q^* = 1$  unit of production.

### 3.4. Deterrent monopoly under sunk costs

Suppose now that establishing a production capacity requires input investments that will be lost even if production does not occur and that can therefore be lost in the process of competition. On this basis we define deterrent monopoly.

Specifically, suppose that each production round for  $X$  takes two periods. In the first period all producers, big and small, buy the inputs for the next period. In the next period they decide whether to dedicate their inputs fully to production or to sell the remaining inputs at a resale price which is a proportion  $0 < k < 1$  of its purchase value, implying a loss representing the sunk cost.

A deterrent monopolist will be one that buys inputs for producing the full quantity  $Q_c^*$  of the competitive case. If further entry into production occurs,

and the deterrent monopolist produces to capacity, the resulting price will be lower than the competitive price  $P_c^*$ . Every producer will loose. If for this reason there in fact is no additional entry, the deterrent monopolist need not produce the full amount  $Q_c^*$ , and can therefore obtain a profit. We examine the resulting game in the next section. For now we calculate the profit that occurs at this point if there is no entry. This results from maximizing

$$\max_Q \pi_D(Q) = TR(Q) - Q_c^* \gamma (Q_c^* - Q) \quad (18)$$

Writing  $Q_D^k$  for the optimum production quantity for the deterrent monopolist, the first order condition yields

$$TR'(Q_D^k) = k\gamma \quad (19)$$

We make the following assumptions on the total revenue function  $TR(Q)$ .

- 1)  $TR'(Q_c^*) < 0$ . This means that revenue can be increased at the competitive equilibrium by reducing the quantity sold.  
And, for simplicity,
- 2) There is a unique  $\bar{Q}$  at which total revenue  $TR(\bar{Q})$  is maximal.
- 3) Marginal revenue is decreasing:  $TR''(Q) < 0$  on  $[0, Q_c^*]$ .

The unique maximum  $\bar{Q}$  therefore satisfies  $0 < \bar{Q} < Q_c^*$ ,  $TR'(Q) > 0$  on  $[0, \bar{Q}]$ , and  $TR'(Q) < 0$  on  $[\bar{Q}, Q_c^*]$ .

The optimal production quantities  $Q_D^k$  chosen by deterrent monopolists are decreasing in the resale rate  $k$  and range monotonically over the interval  $[\bar{Q}, Q_c^*]$ , with  $Q_D^1 = Q_c^*$  and  $Q_D^0 = \bar{Q}$ . Let  $\pi_D^k = \pi_D(Q_D^k)$ . By the envelope theorem  $\frac{d}{dk} \pi_D^k = -(Q_c^* - Q_D^k) \gamma < 0$  profits rise as the optimal quantity  $Q_D^k$  decreases, and this occurs as the resale rate  $k$  increases. Prices  $P_D^k$  move in the opposite direction. Figure 1 shows a price and quantity diagram under deterrent monopoly.

Figure 1. Deterrent monopoly price and quantity levels incorporated in the conventional monopoly diagram. When  $K=0$ , equilibrium sales are  $\bar{Q}$ , where  $MR = 0$ . When  $k=1$  and there are no sunk costs, equilibrium sales are the monopoly level  $Q_M$ . The magnitude of the triangular area below the axis equals the difference in total revenues between deterrent monopoly and

competition. Unit costs for deterrent monopoly and competition are equal since the entry threat is supplying the full market.

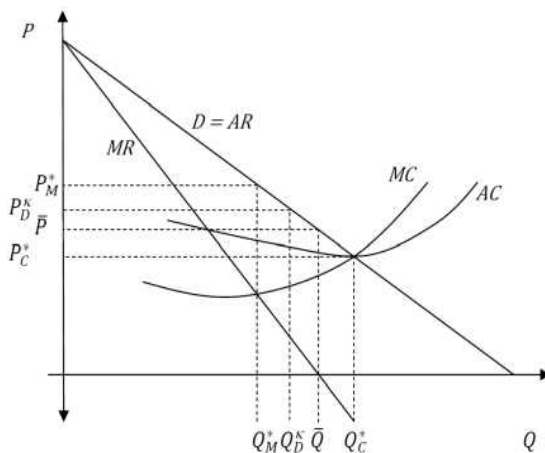


Figure 1

### 3.5. The deterrent monopoly game

We just saw that under the sunk cost conditions we defined, a large producer purchasing inputs for producing the full quantity  $Q_c^*$  can threaten other producers with losses if they enter production. This means that a game for market share can develop. We discuss and construct through several sections the case in which one big producer competes with a continuum of small producers.

The game consists of a repetition of production rounds. Each production round is the stage game of an infinitely repeated game. The players are Big, who is the large producer, and Small, a continuum of small players with  $i \in [0, N_{s_{\max}}]$ , where the  $N_{s_{\max}}$  is larger than  $N_c^*$ , the measure or number of small players needed to supply the competitive market at optimal plant capacity.<sup>4</sup> Once we discuss the competitive case below and establish how the continuum of small players coordinates to participate, the game can be analyzed as a game with two players, Big and Small.

<sup>4</sup> Abusing language, I will use number instead of measure for small producers. Thus NS small producers refers to a set of measure  $N_s$  of small producers, usually  $[0, N_s]$ .

In the first period of each round, Big establishes a continuum of plants  $i \in [0, N_{1B}]$ , each with an optimal unit productive capacity. Then in period 2 she decides to operate  $N_{2B} \leq N_{1B}$  of these plants, at optimal capacity  $q^*=1$ . Remaining inputs are sold at a loss proportional to  $1-K$ . Meanwhile in the first period a continuum  $[0, N_{1S}]$  of small producers each establishes an infinitesimal plant with unit productive capacity.<sup>5</sup> In period 2 some set of small producers  $i \in [0, N_{2S}]$  decide to produce at unit capacity, while the remaining producers  $i \in [N_{2S}, N_{1S}]$  sell their inputs at a loss proportional to  $1-k$ . All inputs will be used for production,  $N_{2S}=N_{1S}$ , when  $P > ky$  (even if  $P < y$ ), because in this case if there is a loss from selling at less than market price it is still less than the loss incurred from reselling inputs. When  $P = ky$ , so both losses are equal, there will be some equilibrium production level  $0 \leq N_{2S} \leq N_{1S}$  and when  $P < ky$ , reselling the inputs is preferable and  $N_{2S}=0$ .

Introduce parameter  $t$  to indicate the round in the repeated game. The actions of players Big and Smalli in the stage game are parametrized by  $(N_{1B}^t, N_{2B}^t), (N_{1S}^t, N_{2S}^t)$ . Quantities in the first period of each round  $t$  represent the quantities for which inputs were bought, while quantities in the second period represent the actual quantities produced.

### 3.5.1. The stage game

The main asymmetry between small players and Big is size, and the implication that size has on strategic play. Small players cannot affect prices, because the quantities they produce are too minute, and therefore do not affect Big. Only the non-cooperative collective decisions taken by all small players together affect Big. It is nevertheless possible for small players to have strategies, as we shall see below.

The stage game of our infinitely repeated game consists of the two period rounds that we have described, written as if they occurred at time  $t$ . We write the game as if there were two players, Big and Small, where the actions taken by Small are the result of the actions of all small players. The actions available to each player are:

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<sup>5</sup> For simplicity we assume that the small, infinitesimal producers are of equal size. It would not be too difficult to introduce a measure representing different sizes.

$$\left\{ \left( N_{1B}^t, N_{2B}^t \right) : 0 \leq N_{2B}^t \leq N_{1B}^t \leq N_c^* \right\} \quad (20)$$

$$\left\{ \left( N_{1B}^t, N_{2B}^t \right) : 0 \leq N_{2B}^t \leq N_{1S}^t \leq N_S \max \right\} \quad (21)$$

We set  $N_c^*$  as the upper limit for  $N_{1B}^t$ , and also usually for  $N_{1S}^t$ , because there is no incentive to set up productive capacities above levels for which the profit is certain to be zero (something we verify in the exposition below when relevant). The stage game payoffs are as follows. Big's payoff  $\pi_B^t$  is

$$\pi_B^t = P(N_{2B}^t + N_{2S}^t)N_{2B}^t - \gamma N_{1B}^t + k\gamma(N_{1B}^t - N_{2B}^t) \quad (22)$$

where  $P^t = P(N_{2B}^t + N_{2S}^t)$  is the price of good  $X$ , which depends on Big's own production and on the aggregate production of small producers. The discount rate between periods 1 and 2 is assumed to be negligible. The payoff obtained by small player  $i \in [0, N_{1S}^t]$  in period 2 depends on whether she produces or not,

$$\pi_s^t(i) = \begin{cases} P(N_{2B}^t + N_{2S}^t) - \gamma, & i \in [0, N_{2S}^t], \\ -(1-k)\gamma, & i \in (N_{2S}^t - N_{1S}^t) \end{cases} \quad (23)$$

If some small players produce and others do not then these two quantities will be equal.

### 2.5.2. Small players with competitive market strategy

One of the properties of perfect markets is that they generate coordination (Adam Smith's invisible hand). Each producer observes prices and decides how much to supply according to her production possibilities, and an efficient equilibrium is generated. Once producers know the price with certainty, knowledge about the strategies of others, or past "play of the game" is irrelevant. We begin by observing how perfect competition works out in our model when only small players participate, and then sequentially construct the new strategic situations that develop when Big participates.

Because we assume identical small producers, a coordination problem arises in deciding which producers participate and which do not. When for example we say that  $N_c^*$  is the number of small players that clears the

competitive market, and mean that small producers  $i \in [0, N_c^*]$  each produce one unit, while potential participants  $i \in [N_c^*, N_{\text{smax}}]$  do not participate in production, implicitly we are assuming that small players take their decisions instantaneously but in increasing order along the interval  $[0, 1]$ . We make this assumption for how decisions are taken by the small agents. For example, it could be that as  $i$  increases, producers are negligibly less productive, or are informed and commit to production negligibly later.

### 3.5.3. Perfect markets

Consider the case when there are only small producers, so  $N_{1B}^t = N_{2B}^t = 0$ . The  $i^{\text{th}}$  player therefore has the strategy “purchase inputs to produce a unit of  $X$  if the expected price  $P(i)$  is at least the production cost  $y$ , otherwise do not purchase inputs”. Recall that when this producer decides to participate the whole interval  $[0, i]$  of producers is participating. Since the inverse demand function  $P(i)$  is decreasing in  $i$ , the measure of small participants purchasing inputs in period 1 to produce one unit of  $X$  in period 2 is  $N_{1s}^t = N_c^*$ . When period 2 arrives, all of the small producers know that the price  $P$  cannot fall below  $y$ , so none decides to curtail production and  $N_{2s}^t = N_{1s}^t = N_c^*$ . Hence

$$\pi_s^t(i) = P(N_{1s}^t) - y = 0, \quad i \in [0, N_{1s}^t] \quad (24)$$

This is a Nash equilibrium. As we saw, none of the producers has an incentive to change their decision in period 2, so Small has no incentive to change  $N_{2s}^t$ . As for period 1, if  $M_{1s}^t$  producers participate instead of  $N_{1s}^t$ , and  $M_{1s}^t < N_{1s}^t$ , then producers  $(M_{1s}^t, N_{1s}^t]$  are not participating. Producers  $[0, M_{1s}^t]$  obtain positive profits  $1 - M_{1s}^t - y > 0$  which deviating players  $(M_{1s}^t, N_{1s}^t]$  are forsaking to earn 0 instead. If instead  $M_{1s}^t > N_{1s}^t$ , profits would be negative. Hence there are no incentives for new entrants either.

### 3.5.4. Big comes in

Now consider the case when small producers keep to the competitive market strategy but a large producer also participates in the market. Suppose the large player, Big, sets up in period 1 a productive capacity sufficient to supply the full competitive market but then in period 2 underproduces to raise the



price. If small producers keep to their original competitive strategy, they will not purchase any inputs and Big will make a profit unimpeded.

However, there is a new element to strategy here. Big is no longer acting exclusively according to expected price. She is elbowing out other participants. The way this occurs is that her intended productive capacity, as measured by purchased inputs, remains fixed independently of the intentions of small agents. We can model this by supposing that Big announces her productive capacity before the small agents, or by supposing that all players have a last ditch opportunity to cancel their input purchases after everybody announces their intentions. Big refuses to cancel and thus small players cancel instead. Below we model this properly as equilibria in a game. The point however is to note that by including knowledge of other player's strategies we are actually starting to go beyond the simple, innocent competitive market strategy.

In this protogame, for any announcement  $N_{1B} \leq P^{-1}(y) = N_c^*$  made by Big,  $N_{1s} = N_c^* - N_{1B}$  small producers purchase inputs to produce a unit of good  $X$  in period 2. Then if Big increases the price by reducing production, small agents produce to full capacity and Big maximizes

$$\pi_B(N_{1B}, N_{1s}, N_{2B}) = P(N_{1B} + N_{2B})N_{2B} - \gamma N_{1B} + k\gamma(N_{1B} - N_{2B}) \quad (25)$$

Define the total revenue function

$$TR(N_{1s}, N_{2B}) = P(N_{1s} + N_{2B})N_{2B} \quad (26)$$

For simplicity we assume this function, and therefore also  $\pi_B$ , is concave in  $N_{2B}$  for any  $N_{1s}$ . That is,

$$\frac{\partial^2 TR}{\partial N_{2B}^2}(N_{1s}, N_{2B}) = P''(N_{1s} + N_{2B})N_{2B} + 2P'(N_{1s} + N_{2B}) < 0 \quad (27)$$

We can now determine optimal production levels for Big for different levels of small participants.

**Lemma 1.** Optimal Production Levels. Suppose that inputs purchased for production by Big and Small do not on their own oversupply the market, but jointly cover or exceed the competitive level of supply, that is,  $N_{1B} \leq N_c^*$ ,  $N_{1s} \leq N_c^*$  and  $N_{1B} + N_{1s} \geq N_c^*$ . There is some maximal number of small participants  $\bar{N}_{1s} \in (0, N_c^*)$  for which when  $N_{1s} < \bar{N}_{1s}$  Big's optimal production level is to undersupply the market by choosing  $N_{2B}^* \in (0, N_c^* - N_{1s})$ . In these cases, if  $N_{1B} + N_{1s} = N_c^*$ , small

players will dedicate all of their inputs to production, setting  $N_{2S} = N_{1S}$ , and will obtain a proportionally higher profit than Big, freeriding on her underproduction. If instead  $N_{1S} > \bar{N}_{1S}$ , Big's optimal response always yields a negative payoff unless  $N_{1B} + N_{1S} = N_c^*$ , in which case all inputs are assigned to production, so  $N_{2B} = N_{1B}$  and the payoff is zero for both Big and Small. When input purchases imply oversupply, so  $N_{1B} + N_{1S} > N_c^*$ , payoffs remain invariant if all input resales are carried out by Big, so  $N_{2S} = N_{1S}$  can be assumed. In the case  $N_{1S} = \bar{N}_{1S}$ , the optimal response  $N_{2B} = N_c^* - N_{1S}$  yields competitive market prices. Big's payoffs are decreasing in Small's participation and in her own initial input purchase.

**Proof.** Big's optimization problem is

$$\pi_B^*(N_{1B}, N_{1S}) = N_{2B} \leq N_{1B} \quad \pi_B(N_{1B}, N_{1S}, N_{2B}) \quad (28)$$

where (25) defines  $\pi_B$ . Observe that on line  $N_{1S} + N_{2B} = N_c^*$ , Big's marginal profit is:

$$\left. \frac{\partial \pi_B}{\partial N_{2B}} \right|_{N_{2B} = N_c^* - N_{1S}} = P'(N_c^*) N_{2B} + \gamma - \gamma + (1-k)\gamma \quad (29)$$

$$= P'(N_c^*) (N_c^* - N_{1S}) + (1-k)\gamma \quad (30)$$

$$\begin{cases} < -P'(N_c^*) + (1-k)\gamma = -k\gamma < 0 & \text{for } N_{1S} = 0 \\ = (1-k)\gamma > 0 & \text{for } N_{1S} = N_c^* \end{cases} \quad (31)$$

In addition, the expression is monotonic, since

$$\frac{d}{dN_{1S}} \left( \left. \frac{\partial \pi_B}{\partial N_{2B}} \right|_{N_{2B} = N_c^* - N_{1S}} \right) = -P'(N_c^*) > 0 \quad (32)$$

Hence  $\left. \frac{\partial \pi_B}{\partial N_{2B}} \right|_{(N_{1B}, \bar{N}_{1S}, N_c^* - \bar{N}_{1S})} = 0$  defines a unique value  $N_{1S} \in (0, N_c^*)$  below which  $\frac{\partial \pi_B}{\partial N_{2B}} < 0$  and above which  $\frac{\partial \pi_B}{\partial N_{2B}} > 0$ . Since on line  $N_{2B} = 0$ ,  $\left. \frac{\partial \pi_B}{\partial N_{2B}} \right|_{N_{2B}=0} = P(N_{1S}) - k\gamma > P(N_c^*) - k\gamma = (1-k)\gamma > 0$ , for  $N_{1S} < \bar{N}_{1S}$  Big's maximum payoff occurs with undersupply, and for  $N_{1S} > \bar{N}_{1S}$  no undersupply is worthwhile. In the case when  $N_{1B} > N_{1S} = N_c^*$  setting  $N_{2B} = N_{1B}$  yields a zero payoff. Hence when  $N_{1S} < \bar{N}_{1S}$  profits  $\pi_B^*(N_{1B}, N_{1S})$  are positive. Consider now the case when  $N_{1S} + N_{2B} > N_c^*$ .

Because  $\frac{\partial \pi_B}{\partial N_{2B}}$  is independent of  $N_{1B}$ , if  $N_{1S} \leq \bar{N}_{1S}$ , the optimal  $N_{2B}$  remains unchanged. If instead  $N_{1S} > \bar{N}_{1S}$ ,  $\left. \frac{\partial \pi_B}{\partial N_{2B}} \right|_{N_{2B} = N_c^* - N_{1S}} > 0$  implies Big's optimal payoff is

negative. In this case, if at Big's optimal oversupply level selling the product is still better than reselling the inputs, so  $P(N_{1s} + N_{2B}) > ky$ , small producers set  $N_{2s} = N_{1s}$ . Otherwise Big limits overproduction to set  $P(N_{1s} + N_{2B}) = ky$ . Big can always do this, because  $N_{1s} \leq N_c^*$ . Since at this price selling the product or reselling the inputs yields the same payoffs, we can assume  $N_{2s} = N_{1s}$ , so only Big resells inputs. Finally, by the envelope theorem that  $\frac{\partial \pi_B^*}{\partial N_{1s}}(N_{1B}, N_{1s}) = P'(N_{1s} + N_{2B})N_{2B} < 0$  and  $\frac{\partial \pi_B^*}{\partial N_{1B}}(N_{1B}, N_{1s}) = -(1-k)\gamma < 0$ .

It follows that if the number of small participants is less than  $\bar{N}_{1s}$ , Big can still make a profit by undersupplying the market. In this case we say Big holds a *partial deterrent monopoly*. Write:

$$N_{DN}^* = N_{DM}^*(N_{1s}) = N_{2B}^*(N_{1s}) + N_{1s} < N_c^* \text{ if } N_{1s} < \bar{N}_{1s} \quad (33)$$

$$P_{DM}^* = P_{DM}^*(N_{1s}) = P(N_{DM}^*(N_{1s})) > \gamma \text{ if } N_{1s} < \bar{N}_{1s} \quad (34)$$

$$\pi_{BDM}^* = \pi_{BDM}^*(N_{1s}) = \pi_B(N_c^* - N_{1s}, N_{1s}, N_{2B}^*(N_{1s})) \quad (35)$$

for the total quantity supplied to the market in this case, Big's optimal response in period 2, the resulting price, and Big's profit when  $N_{1B} = N_c^* - N_{1s}$ . "DM" means "deterrent monopoly" and refers to the partial case when  $N_{1s} > 0$ . The dependence on  $N_{1s}$  will be omitted from the notation unless required, and primes used for derivatives. Big's profit is positive if the input purchase does not already oversupply the market, that is,  $\pi_{BDM}^* > 0$ . In this case the small participants make a proportionally higher profit, because they do not spend on deterrence by reselling their inputs at a loss.

Proposition 6 in the appendix shows that as the number  $N_{1s}$  of small free riders increases, the aggregate quantity  $N_{DM}^*$  produced in partial deterrent monopoly increases and so the equilibrium price decreases.

In the deterrent monopoly game below, Big will dissuade small participation by threatening to oversupply the market and causing everybody a loss. What is Big's optimal production choice in this case? Suppose  $N_{1s} + N_{1B} \geq N_c^*$ . Then Big's optimization problem for punishing is:

$$\pi_{BDM}^* = \pi_{BDM}^*(N_{1s}) = \pi_B(N_c^* - N_{1s}, N_{1s}, N_{2B}^*(N_{1s}))$$

The only change from Lemma 1 is the restriction  $P(N_{1s} + N_{2B}) \leq y$ .

**Corollary 2.** Cheapest Punishment. Suppose that inputs purchased for production by Big and Small do not on their own oversupply the market, but can jointly oversupply the market, that is,  $N_{1s} \leq N_c^*$ ,  $N_{1s} \leq N_c^*$  and  $N_{1B} + N_{1s} > N_c^*$ .

When  $N_{1S} \leq \bar{N}_{1S}$ , Big's highest payoff inflicting punishment occurs with slight oversupply, that is  $N_{2B} = (N_C^* - N_{1S})^+$ .

If instead  $N_{1S} > \bar{N}_{1S}$ , optimal response already produces a negative payoff for both players. Therefore the immediate gain  $\pi^{DEV}$  that Big obtains from deviating from punishment is independent of  $N_{1B}$ , is decreasing in  $N_{1S}$ , and equals.

$$\begin{aligned}\pi^{DEV}(N_{1S}) &= \pi_B^* - \pi_B^{punish} \\ &= \begin{cases} (P(N_{DM}^*) - \gamma)N_{DM}^* - \frac{(1-K)\gamma(N_{DM}^* - N_C^*)}{0} > 0 & N_{1S} < \bar{N}_{1S} \\ 0 & N_{1S} \geq \bar{N}_{1S} \end{cases} \end{aligned} \quad (36)$$

*Proof.* The result follows from the signs for  $\left. \frac{\partial \pi_B}{\partial N_{2B}} \right|_{N_{2B}=N_C^*-N_{1S}}$  established in the proof of the previous lemma. Note for reference that

$$\begin{aligned}\pi_{BDM}^* - \pi_B^{Punish} &= \\ &= P(N_{DM}^*)(N_{DM}^* - N_{1S}) - \gamma(N_{DM}^* - N_{1S}) - (1-K)\gamma(N_{1B} - (N_{DM}^* - N_{1S})) \\ &\quad - \left\{ P(N_C^*)(N_C^* - N_{1S}) - \gamma(N_C^* - N_{1S}) - (1-K)\gamma(N_{1B} - (N_C^* - N_{1S})) \right\} \\ &= \left( P(N_{DM}^*) - \gamma \right) (N_{DM}^* - N_{1S}) - (1-K)\gamma(N_C^* - N_{DM}^*) > 0 \end{aligned} \quad (37)$$

Because  $\pi_B^{Punish} < \pi_B^*$  by construction,  $\gamma < P(N_{DM}^*)$ ,  $N_{DM}^* - N_{1S} = N_{2S}^* > 0$  and  $N_{DM}^* < N_C^*$ .

### 3.5.5. Small gets wise

What happens when small producers observe Big will increase prices in period 2 and make a profit? Small producers must now decide to abandon the simple rules of participation in perfectly competitive markets, to become strategic players. Now both types of players observe each other's strategies. Small producers have some incentives to participate in production, because even though Big purchases inputs to supply the full competitive market, if she then undersupplies the market, small participants will also make a profit. Therefore, Big must threaten to punish small producers with market oversupply if they participate. To show Big's threat is credible, I prove it forms part of a subgame perfect Nash equilibrium in an infinitely repeated game with perfect monitoring.

Recall we assume that after Big has purchased inputs, small agents have a last chance to purchase inputs as well. Hence if at any time Big does not

purchase enough inputs in the first period to supply the full competitive market, small agents will be aware of this and purchase inputs to complete this supply, making at least the normal profit (zero) in any circumstance.

Big's strategy for making a profit depends on threatening small players with low prices if they participate. A strategic response that is available to small players is to participate no matter what, rendering Big's threat meaningless. In equilibrium Small and Big's strategies will adjust to each other. Corresponding to some minimum intended participation  $N'$  on Small's part, Big will have some toleration  $T'$  for participation of small players whom she will not punish, instead diminishing her own participation. However, if Big shirks from punishing, as a consequence Small will in the next round raise her intended participation  $N^{t+1}$ , forcing Big to raise her toleration level  $T^{t+1}$ . In a Nash equilibrium the sequences  $T'$  and  $N'$  will be equal. For if  $T' > N'$ , Big will not be taking full advantage of her possible share of production and will therefore profit by raising  $T'$ . Likewise Small will have incentives to raise  $N'$ . If instead  $T' < N'$ , punishment is sure to follow and both Small and Big will have incentives to deviate from their strategies.

Consider strategies in which Small raises next round's intended participation when a) Small behaves within Big's toleration level but Big does not purchase the full share of inputs  $N_c^* - T'$  that Small has conceded to her, therefore establishing a higher level of toleration, or b) Small exceeds Big's toleration levels and Big fails to punish Small, in which case Small raises her intended participation by some measure  $\eta$ , Small's "response parameter."

There are several ways in which the coordination amongst small players making these strategies possible may be thought to occur. One alternative is to think that small players are graded according to toughness. If they see that Big does not take full advantage of her market share or shirks on her punishment, the toughest set of small players will decide to react, and this set will be larger the more Big has deviated. An alternative is to think that all small players react identically and assign a higher uniform probability  $N^{t+1}/N_{s_{max}}$  to participate. An independent grading in productivity can still subsist in both cases.

Big has corresponding strategies for changing her toleration level  $T'$ . She diminishes  $T'$  if Small players do not take full advantage of their possibilities  $N'$ . Also, she knows she must raise  $T'$  if she shirks from punishment.

Big's toleration level  $T'$  and Small's minimum intended participation level  $N'$  carry all of the relevant information on the past play of the game for

defining current strategy. The selection of  $N^t$  and  $T^t$  may be thought to occur in a period 0, before the selection of  $(N_{1B}^t, N_{2B}^t), (N_{1S}^t, N_{2S}^t)$ .

The joint dynamics of  $T^t$  and  $N^t$  represent a power play between Big and Small which transcends the supply and demand schedules for good  $X$ . So long as the nature of this power play is not fully defined, the deterrent monopoly game will support multiple equilibria. However, this fact in itself already stands in the realm of imperfect markets and for the purposes of the present article is therefore sufficient. The power play may differ in different circumstances. The model of vulnerable markets presented here is therefore open to specification under different circumstances determining the relative power of the agents. The result of the model will be that in the presence of a large player, multiple equilibria arise whose selection depends on a power play between Big and Small that transcends the economic characteristics of good  $X$ . Thus the presence of a large player makes the market imperfect.

The underlying mathematics bring up a basic dichotomy. For one type of equilibrium, demopoly, the economic characteristics of good  $X$  are sufficient to uniquely determine equilibrium price and quantity. For the other type of equilibrium, imperfect markets, in this case deterrent monopoly, while we know equilibrium prices will be higher and equilibrium quantities lower, additional information describing the power relationship between the agents is required to uniquely determine an equilibrium.

### 3.5.6. Subgame perfect strategies

Summarizing, consider the following strategies for Big and Small.

Period 0. Toleration and minimum participation levels are defined by initial values  $T^t, N^t$  according to the following dynamics:

$$T^{t+1} = \begin{cases} \min\{N^t, N_{1S}^t\} & \text{if } N_{1S}^t \leq T^t, \\ \min\{\max\{N^t, N_{1S}^t\} + \eta_c N_c^* & \text{if } N_{1S}^t \leq T^t \text{ and } \\ T^t & N_{1S}^t + N_{2B}^t < N_c^*, \\ & \text{otherwise;} \end{cases} \quad t = 1, 2, \dots \quad (38)$$

$$\begin{aligned}
N^{t+1} = \begin{cases} \max\{N^t, N_c^* - N_{1B}^t\} & \text{if } N_{1s}^t \leq T^t, \\ \min\{\max\{N^t, N_{1s}^t\} + n, N_c^*\} & \text{if } N_{1s}^t \leq T^t \text{ and} \\ & N^t \\ & N_{1s}^t + N_{2B}^t < N_c^*, t = 1, 2, \dots \\ & \text{otherwise;} \end{cases} \quad 39
\end{aligned}$$

Recall that for the subgame perfect Nash equilibrium  $T^t = N^t$ .

What will be shown is that for any response parameter  $\eta > 0$  and for high enough discount rate  $\delta$  approaching 1, there is a critical level  $C_\delta \in (0, \bar{N}_{1s})$  for toleration and intended participation, below which punishment is a credible threat and above which profits become too low to warrant a credible threat. Recall  $\bar{N}_{1s}$  is the number of small free riders at which Big's profits reduce to zero. A strategy pair for Big and Small with initial intended participation and toleration levels  $N^t = T^t < C_\delta$  will yields a Nash equilibrium with that level of small participants. However, if  $N^t = T^t$  ever rises above the critical level  $C_\delta$ , the number of small participants will immediately rise to at least  $\bar{N}_{1s}$  because at levels in  $(C_\delta, \bar{N}_{1s})$ , Big is sure to shirk punishment, undersupply and make a profit, so that additional plants set up by small producers will also make a profit. Hence for intended participation levels  $N^t = T^t \in (C_\delta, N_c^*]$  above the critical level, there will be a competitive Nash equilibrium with participations  $N_{1s} = \max\{N^t, \bar{N}_{1s}\}$ ,  $N_{1B} = N_c^* - N_{1s}$ .

Periods 1 and 2. Production strategies. For  $N^t, T^t \leq C_\delta$ ,

$$\begin{aligned}
N_{1B}^t &= N_c^* - T^t, \\
N_{1s}^t &= \max\{N_c^* - N_{1B}^t, N^t\}, \\
N_{2B}^t &= \begin{cases} N_{2B}^*(N_{1s}^t) (\text{undersupply}) & \text{if } N_{1s}^t \leq T^t \\ N_c^* - N_{1s}^t (\text{punishment}) & \text{if } N_{1s}^t > T^t \end{cases} \quad (40)
\end{aligned}$$

Note that strategy in period 1 is written with Small having a last round of decision after Big in case Big underinvests. Similarly, period 2 is written as a function of any arbitrary play in period 1 satisfying  $N_{1s}^t + N_{1B}^t \geq N_c^*$  (because otherwise even competitive small players are loosing an opportunity to participate). This makes it possible to begin the game at any period of the stage game, as required in the evaluation of one-shot deviations. Note that  $N^t$  and  $T^t$  need not be equal in principle. Small and Big's strategies are functions of  $N^t$  and  $T^t$  independently. We saw in Lemma 1 and its Corollary that whether



Big punishes or undersupplies to make a profit, Small can be considered to choose  $N'_{2s} = N'_{1s}$ .

Finally, for  $N^t$ ,  $T^t > C_\delta$ , letting Small have a last round of decision after Big in case she underinvests,

$$\begin{aligned} N_{1B}^t &= N_C^* - \max\{T^t, \bar{N}_{1s}\}, \\ N_{1s}^t &= \max\{N_C^* - N_{1B}^t, N^t, \bar{N}_{1s}\}, \\ N_{2B}^t &= N_{1B}^t, \\ N_{2s}^t &= N_{1s}^t \end{aligned} \tag{41}$$

While the game is written as a two player game, player Small reflects the aggregate behavior of the continuum of small players Smalli  $i \in [0, N_{s \max}]$ , according to the coordination rules described for perfect competition, extrapolated to also coordinate strategy choices.

**Theorem 3.** Let  $N^1 = T^1$  take any value on  $[0, N_C^*]$ . Strategies (38), (39), (40) and (41) define a subgame perfect Nash equilibrium. In each of these equilibria Big and Small's play remains constant in each round  $t$ . In particular  $N'_{2s} = N'_{1s}$  are constant and lie in the disjoint union  $[0, C_\delta] \cup [N_{1s}, N_C^*]$ . Equilibrium can only take place in the first interval if Big can purchase production inputs for at least  $N_C^* - C_\delta$  plants. In this case the choice of solution interval, and the choice of equilibrium in the first interval, depends on a power play between Big and Small that is independent of the supply and demand of good  $X$ . The first solution interval parameterizes a family of partial deterrent monopolies with price above marginal cost, held by Big with some profitable participation by small agents. The second solution interval parameterizes a family of demopolies in which Big may participate partially but without market power. In this interval the unique equilibrium price equals marginal cost.

The proof is in the subsections that follow. It is based on Ray's (2003) summary of the one-shot deviation principle. We first write down the payoffs, then examine possible deviations within the stage game, and finally show that there are no profitable oneshot deviations.

### 3.5.7. Payoffs

The payoffs for stage  $t$  of the game given initial  $N^t = T^t$  are the following. Solong as  $N^t \leq C_\delta$ , Big's payoff is  $\pi_{BDM}^*(N^t) > 0$ , see (35), and each Small's; payoff is

$P(N_{DM}^*) - y > 0$ , see (34). If instead  $N^t > C_\delta$ , all payoffs are 0. The payoff of the infinitely repeated game is  $(1-\delta)^{-1}$  times these payoffs.

### 2.5.8. Deviation payoffs

We now examine stage  $t$  of the game for differences in payoffs that might be relevant for our examination of one-shot deviations below. First we examine deviations by Big.

In period 0, Big can deviate by changing  $T^t$  to  $T^{t'}$ . Suppose she raises her toleration. The only relevant case is  $T^t < C_\delta$ , because otherwise all payoffs are 0. She will then lower her input purchase  $N_{Bt}^t$ . Therefore Small will correspondingly raise  $N_{1s}^t$  to  $T^{t'}$ . This will lower Big's immediate profits. Small will also permanently raise her intended participation level  $N^{t+1}$  to  $N_C^* - N_{1B}^t = T^{t'}$ , and Big will also raise  $T^{t+1}$  to  $T^{t'}$ . Suppose instead she lowers her toleration. Small has not lowered  $N^t$ , so will elicit a punishment from Big, in the current and in all future periods. In both cases Big will loose both immediately and in the future.

Consider periods 1 and 2 and suppose  $N^t = T^t < C_\delta$ . Big can deviate in the first period by changing  $N_{1B}^t$ . If Big attempts to undersupply the market by setting  $N_{1B}^t < N_C^* - N_{1S}^t$ , Small immediately completes its supply with  $N_{1S}^t = N_C^* - N_{1B}^t > N_{1S}^t$ . Big therefore suffers a double loss. First, her optimal stage  $t$  payoff is now lower, because  $\pi_{BDM}^* N_{1S}^t$  is decreasing in  $N_{1S}^t$ . Second, Small will raise her intended participation in the future, lowering Big's payoff.

If instead Big oversupplies small producers by setting  $N_{1B}^t > N_C^* - N_{1S}^t$  (even in the case  $N^t = 0$ ), Small will still purchase production inputs according to  $N_{2S}^t = N_{1S}^t = N^t$ , reducing Big's payoff, which is decreasing in  $N_{1B}^t$ .

Alternatively, Big deviates in the second period, after any given play  $(N_{1B}^t, N_{1S}^t)$ , which now serves as initial point of the game for one shot deviations. Recall we need only consider input purchases satisfying  $N_{1S}^t \geq N_C^* - N_{1B}^t$ , and production decisions  $N_{2S}^t = N_{1S}^t$  by Small. In considering deviations, since we have already chosen optimal play following Lemma 1 and its Corollary, there are only two deviations left to consider, deciding to punish when no punishment is called for, for which there are no incentives since this is costly and has no implications for future play, or deciding not to punish when punishment is called for, which affects future play. This case, which will only present a gain for Big when  $N_{1S}^t < \bar{N}_{1S}$ , is examined as a one-shot deviation below.

Suppose now  $N^t = T^t \geq C_\delta$ . Small's intended participation is now so high that punishment is meaningless. Big can in no circumstance expect a positive profit, so cannot do better than participating with a 0 payoff. Her only alternative is to reduce her participation, something which offers no gain.

Now let us examine deviations by Small. In period 0, suppose Small raises her minimum intention to participate. This will elicit punishment by Big in this and every future period. If instead small players reduce  $N^t$  to  $N^t$ , Big still plays according to her original toleration level  $T^t = N^t$ , so in the last round of decision in period 1 Small will raise her input purchase from  $N^t$  back to  $N^t$ , and end up setting  $N^{t+1} = N^t$ . Thus there are no incentives for this deviation.

In periods 1 and 2, suppose  $N^t \leq C_\delta$  and Small changes  $N^t_{1s}$ . if  $N^t_{1s} < N^t$ , profits increase for Big and for the remaining small participants, but decrease for those small players who have forgone participation, with no gain in the future. If instead  $N^t_{1s} > N^t$ , then Big punishes and all small players experiences a loss, again with no gain in the future. Therefore none of these cases need be considered in one shot deviations for the present game. There are also no profitable deviations for Small in the second period or in the case  $N^t > C_\delta$ .

### 3.5.9. One shot deviations

Our analysis of deviations in the stage game shows that only one profitable candidate for a one shot deviation exists, apart from the possibility of shifting  $C_\delta$ , which we also examine here. This is for Big not to punish in the second round. To see if this is profitable, the immediate gain must be compared with the loss in future play. Suppose therefore  $N^t = T^t \leq C_\delta$ , and small plays  $N^t_{1s} > N^t$ . We saw that Big has one optimal choice for punishing, which consists of just oversupplying the market. The value  $V$  of deviating by not punishing is:

$$V = \left\{ \pi^{DEV}(N^t_{1s}) + \frac{\delta}{1-\delta} \left( \pi^*_{BDM}(N^t_{1s} + n) - \pi^*_{BDM}(N^t) \right) \text{ if } N^t_{1s} + nC_\delta \right. \\ \left. \pi^{DEV}(N^t_{1s}) - \frac{\delta}{1-\delta} \pi^*_{BDM}(N^t) \text{ if } N^t_{1s} + n > C_\delta \right. \quad (42)$$

The first condition is vacuous when  $n > C_\delta$ . For  $V \leq 0$  (we consider there is no deviation if the gain is zero) it is necessary that:

$$\frac{\delta}{1-\delta} \geq N^t \leq N^t_{1s} \leq c_\delta - n\pi^*_{BDM}(N^t) - \frac{\pi^{DEV}(N^t_{1s})}{\pi^*_{BDM}(N^t) - \pi^*_{BDM}(N^t_{1s} + n)} \quad (43)$$

for the first inequality, and

$$\frac{\delta}{1-\delta} \geq N_{1s}^t \geq \max\{C_\delta - \eta, N^t\}, 0 \leq N_{1s}^t \leq C_\delta \frac{\pi^{DEV}(N_{1s}^t)}{\pi_{BDM}^*(N^t)} \quad (44)$$

for the second. Proposition 6 in the appendix shows that  $\pi_{BDM}^*$  is positive, strictly decreasing and convex on  $(0, \bar{N}_{1s})$ , so that the supremum in (43) is:

$$\leq N^t \leq C_\delta - \eta \pi_{BDM}^* \frac{\pi^{DEV}(N^t)}{(N^t) - \pi_{BDM}^*(N^t + \eta)} = \frac{\pi^{DEV}(0)}{\pi_{BDM}^*(C_\delta - \eta) - \pi_{BDM}^*(C_\delta)} \quad (45)$$

$$< \frac{\pi^{DEV}(0)}{\pi_{BDM}^*(\bar{N}_{1s} - \eta) - \pi_{BDM}^*(\bar{N}_{1s})} = \frac{\pi^{DEV}(0)}{\pi_{BDM}^*(\bar{N}_{1s} - \eta)} \quad (46)$$

Therefore for any  $\eta > 0$ , a large enough  $\delta$  approaching 1 will exist satisfying inequality

$$\frac{\delta}{1-\delta} \geq \frac{\pi^{DEV}(0)}{\pi_{BDM}^*(\bar{N}_{1s} - \eta)} \quad (47)$$

and therefore (43). Note the crucial role played by  $\eta > 0$ , meaning that a finite rather than infinitesimal increase of intended participation by Small occurs even if Big has shirked punishment when Small has overstepped the limit  $T^t$  by an arbitrarily small amount.

Now because of the role  $C_\delta$  plays in the choice of strategies (40), (41), we have one more condition. This is that for values  $N^t > C_\delta$ , punishment not be credible. This defines  $C_\delta \in (0, \bar{N}_{1s})$  by replacing inequality (44) with.

$$\frac{\delta}{1-\delta} = \frac{\pi^{DEV}(0)}{\pi_{BDM}^*(C_\delta)} \quad (48)$$

Such a  $C_\delta$  exists so long as  $\delta$  is chosen so that  $\frac{\delta}{1-\delta} > \frac{\pi^{DEV}(0)}{\pi_{BDM}^*(0)}$ .

This completes the proof of Theorem 3.

### 3.6. Two corollaries

We can write down the following corollary on how financial development may diminish competition.

**Corollary 4.** Financial development versus competition. Suppose aggregate financial development is high enough that financial institutions can borrow

resources  $N_C^*$  from small players and lend them to an agent Big at a total transaction cost  $TC$  less than the deterrent profits  $\pi_{BDM}^*(0)$ . Then a deterrent monopoly by Big is possible.

**Proof.** Small players will prefer to lend financial institutions the resources  $N_{IS}^T = N_C^*$  that they would dedicate to production so long as the aggregate payoff  $\pi_s$  offered to them is positive,  $\pi_s > 0$ . A deterrent monopoly by Big is possible so long as the total interest  $R$  she has to pay on these resources satisfies  $0 < R < \pi_{BDM}^*(0)$ . Financial institutions will take care of these transactions so long as  $R > \pi_s + TC$ . Since  $TC < \pi_{BDM}^*(0)$  amounts  $\pi_s$  and  $R$  exist satisfying these restrictions. An analogous statement is possible for partial deterrent monopoly.

In the case of a deterrent monopoly, its payoff minus transaction costs,  $\pi_{BDM}^*(0) - TC$  will be distributed between Big, the financial system and small agents according to the prevailing financial market structure and the agents' bargaining power. As compared to demopoly, the resulting deterrent monopoly will be inefficient because 1) it will supply less than the optimal amount of good  $X$ ; 2) at a higher than optimal price; and 3) it will waste resources on deterrence during every production round. In addition, as compared to demopoly, deterrent monopoly will transfer wealth to the financial system and to Big.

We can also note the following strategy for preventing monopolization of good  $X$  that is available to a government that can produce competitively.

**Corollary 5** The public option. Suppose the government produces in  $C_\delta$  plants, selling at a competitive price. Then Big will not establish a deterrent or partial deterrent monopoly.

**Proof.** Big has no incentives to establish a deterrent monopoly if her maximum market share is less than  $N_C^* - C_\delta$ .

Note that once the government establishes the public option, or subcontracts it to small agents, it is the market that establishes the competitive price.

#### 4. Conclusion

We have shown the theoretical existence of *vulnerable markets* for a wide class of productive contexts. These are markets that can support both competitive equilibria and equilibria with market power, in our case deterrent monopoly (partial or full). In the presence of large agents, the price and

quantity outcomes of these imperfect market equilibria are not fully determined by demand and supply, but instead depend on a power play between small and large participants. The only a priori characteristic distinguishing the competitive and the imperfect market equilibria is the ownership structure, in other words, whether large agents are involved or not. The technologies for production and competition, as well as consumer preferences, remain unchanged. Big agents, of course, have recourse to strategies that are unavailable to small agents and that depend on impacting the whole market.

The existence of vulnerable markets provides incentives for financial institutions to concentrate ownership for profit. As compared to demopoly, the resulting equilibrium is inefficient in that it sets prices higher than marginal cost; waists resources on deterrence; and transfers wealth from small producers and consumers to financial institutions and large agents.

An analysis of the implications of this model for competition policy lies beyond the scope of this article. Nevertheless, it is clear that the participation of small and large producers in the political process can have very significant impacts on market structure. A first result is the following: a policy providing a public option can keep a vulnerable market competitive.

## 5. Appendix

**Proposition 6.** Under partial deterrent monopoly, aggregate supply  $N_{DM}^*$  is increasing in  $N_{1S}$ . Big's profits  $\pi_{BDM}^*$  are positive, strictly decreasing and convex on  $[0, \bar{N}_{1S})$  and equals zero at  $\bar{N}_{1S}$ .

**Proof.** Differentiating the first order condition  $\frac{\partial \pi_B}{\partial N_{2B}} = P'(N_{1S} + N_{2B})N_{2B} + P(N_{1S} + N_{2B})K_Y = 0$

$$0 = P''(N_{1S} + N_{2B}) \left( 1 + N_{2B}^{*'} + P'(N_{1S} + N_{2B}) 1 + 2N_{2B}^{*'} \right) \quad (49)$$

Hence, using  $N_{DM}^{*'} = N_{2B}^{*'} + 1$ , see (33),

$$0 = P''(N_{DM}^*) N_{DM}^{*'} N_{2B}^* + P'(N_{DM}^*) (2N_{DM}^{*'} - 1) \quad (50)$$

Thus, in view of (27),

$$N_{DM}^{*''} = \frac{P''(N_{DM}^*)}{P''(N_{DM}^*) N_{2B}^* + 2P'(N_{DM}^*)} > 0 \quad (51)$$

Using the last two statements in the proof of Lemma 1,

$$\begin{aligned}
 \pi_{BDM}^*(N_{1S}) &= \frac{d}{dN_{1S}} \pi_B^*(N_C^* - N_{1S}, N_{1S}) \\
 &= -\frac{\partial \pi_B^*}{\partial N_{1B}} + \frac{\partial \pi_B^*}{\partial N_{1S}} (N_C^* - N_{1S}, N_{1S}) \\
 &= (1-K)\gamma + P'(N_{DM}^*)N_{2B}^*
 \end{aligned} \tag{52}$$

Therefore, again using (27),

$$\begin{aligned}
 \pi_{BDM}^{*''}(N_{1S}) &= P''(N_{DM}^*)N_{DM}^*N_{2B}^* + P'(N_{DM}^*)N_{2B}^* \\
 &< -2P'(N_{DM}^*)N_{DM}^* + P'(N_{DM}^*)(N_{DB}^* - 1) \\
 &= -P'(N_{DM}^*)N_{DM}^* - P'(N_{DM}^*) \\
 &= -P'(N_{DM}^*)(1 + N_{DM}^*) > 0
 \end{aligned} \tag{53}$$

Now,  $\pi_{BDM}^* > 0$  for  $N_{1S} < \bar{N}_{1S}$ , and  $\pi_{BDM}^*(\bar{N}_{1S}) = 0$ . Hence  $\pi_{BDM}^{*'}(\bar{N}_{1S}) \leq 0$ . and so it follows by the convexity just proved,  $\pi_{BDM}^{*''} > 0$ , that  $\pi_{BDM}^{*'}(\bar{N}_{1S}) < 0$  for  $N_{1S} < \bar{N}_{1S}$ .

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