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RECIPROCAL DUMPING AND ENVIRONMENTAL TAXES

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Resumen

En este trabajo se calcula el impuesto óptimo a la emisión de contaminantes en competencia oligopolística y en condiciones de dumping recíproco, en el que las empresas cuentan con la tecnología adecuada para disminuir la contaminación y poder decidir la cantidad de emisiones generadas. En este modelo, el impuesto óptimo depende principalmente de la cantidad de la desutilidad marginal de contaminar, además del costo de abatimiento.
Palabras clave: dumping recíproco, impuestos, política medioambiental, competición oligopolística
Clasificación JEL: Q52, Q56, F18

Abstract

This paper calculates the optimal tax of the emission of polluting agents in oligopolistic possess and under conditions of the reciprocal dumping, in which the firms count on the appropriate technology to decrease the pollution and

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can decide the amount of emissions generated. In this model the optimal tax mainly depends on the amount of the marginal disutility to pollute, as well as the abatement cost.
Keywords: reciprocal dumping, taxes, environment policy, oligopolistic competition
JEL Classification: Q52, Q56, F18

1. Introduction

Among the instruments of environmental policy more used by the governments to regulate the emissions of polluting agents to the atmosphere we find the taxes.[1] Due to the fact that pollution is a public bad, this can not be corrected by the ordinary mechanisms of the market. Government intervention is required to impose a system of shadow prices for pollution to reduce it; in other words, the agents must pay a price for each unit of emitted pollution. In this way, the externality is corrected, at least in theory. Such a price for pollution is implemented by the government through a tax, whose intention is to make firms pollute reasonably less, since to pollute will become expensive by the tributary cost to be paid by the polluting agents.

The present work analyses the implementation of a tax on the amount of pollution emitted by the firms under conditions of the reciprocal dumping. This paper will deal with an apparently paradoxical situation of international trade: the reciprocal dumping.[2] Under such conditions commerce between two countries in completely identical goods exists (and that in the absence of comparative advantage and with constant yields on scale).[3] And although the force of monopolies and oligopolies is reduced due to the increase of in competition, since the penetration in a country by foreign firms tends to reduce power positions of the local firms, so that the prices of the goods fall more and more approaching the average production costs, it still remains far away from a perfect competition situation of perfect competition.

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1 In the classic work of Cropper and Oates (1992) a more detailed explanation of this instrument of environmental policy can be seen.

2 For more detailed information of the reciprocal dumping model, see Brander (1981), Brander and Krugman (1983), Venables (1985), among others.

3 The most general case of similar goods is known as intraindustry trade, where they emphasize works like Balassa (1966), Krugman (1979) and Lancaster (1980).

This way we developed a Cournot model of oligopolistic competition, which is a partial equilibrium analysis, of reciprocal dumping conditions between two small countries, the typical situation faced by developing economies.[4] Both countries produce the same homogenous good under constant economies of scale and different marginal costs of production. We consider in addition that in both countries there are domestic firms that assign part of their production to the local consumption and leave the rest to the export market.[5] We also suppose that the markets are segmented since constant scale economies exist and there are no capacity restrictions, which imply that for the firms the variations in one of the markets do not influence the decisions taken in the other. The firms generate pollution in their productive processes, yet, they own the proper technology to oppose it, so they can decide the magnitude of the generated pollution; in addition, there exists a social cost to pollute. Under these circumstances the government tries to persuade the firms that they pollute the least possible through the imposition of a tax by unit of emitted pollution. The proposed model calculates the optimal pollution tax that maximises the welfare in each one of the countries. And therefore, the aforementioned optimal tax determines the application of strategic policies under specific conditions that are related to the structure of costs of the firms, specifically the amount of the abatement cost by unit of pollution and its relation with the marginal disutility of polluting. These policies have important consequences in function of social welfare in both countries that involve the consumer surplus, the benefit of the firms and the social cost to pollute.

For this model we concluded that if the marginal cost of polluting is very high, then the government imposes a positive pollution tax that consequently forces the firms to pollute less or to pay it. But if the marginal cost to pollute is not sufficiently large, then the magnitude of the tax depends on the size of the market of the foreign country with respect to the local one. If the first is very big, then the government will try to favour the competitiveness of the local firms establishing a null burden to the emissions of polluting agents, but if the second is much larger, then the principle will prevail to reduce the emission of polluting agents through a tax greater than zero.

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4 Some works that analyse the instruments of environmental policy in oligopolistic models, including some specific of reciprocal dumping, we have: Barret (1994), Espinosa and Palomera (2003), Lahiri and Ono (1998), Espinosa and Ozgur (2001).

5 Even though there are asymmetries in the cost structure, both countries will import the same homogeneous good as a result of strategic decisions of the firms, Brander (1981).

The structure of this work is the following: It begins with the specification and boundary of the model. Next the optimal pollution tax is determined. Finally, from these results applicable environmental policies are set forth.

2. Specification of the model

We consider the trade of a homogenous good between two countries A and B, under conditions of reciprocal dumping. Country A produces for local consumption and to export to country B. Therefore, the production of a particular firm from country A of the homogenous marketable good is:

$$X = X_A + X_B \tag{1}$$

Where, X_A is the quantity of the produced good for local consumption in the country A; X_B is the quantity of the produced good for export to the country B.

Similarly, for country B,

$$Y = Y_A + Y_B \tag{2}$$

Where, Y_B is the quantity of the produced good for the local consumption for the country B and Y_A is the quantity of the produced good for export to the country A.

We can make the assumption that there exist n firms in the country A, and m firms in the country B; in order that the demand in the country A, D_A, is equal to the production for the local consumption combined in their n firms, plus the assigned production to the exports combined of the m firms of country B, this is,

$$D_A = nX_A + mY_A \tag{3}$$

In the same way, the demand in country B will be

$$D_B = mY_B + nX_B \tag{4}$$

We can assume that both countries have the proper technology to regulate their emissions of pollutants. Let z_A be the quantity of pollution per unit produced of the homogeneous good in the country A and let z_B be the quantity of pollution per unit produced of the good in the country B.

Therefore, the quantity of polluting emissions in the country A, z_A, is equal to the total production of the homogeneous good in the country A, given by the production by each domestic company, times the number of firms that participate in the market of the country A, times the quantity of pollution emitted per unit of product, z_A, *i.e.*,

$$Z_A = z_A\left(n\left(X_A + X_B\right)\right) = nX_A z_A + mX_B z_A \qquad (5)$$

In the same way for country B,

$$Z_B = z_B\left(m\left(Y_A + Y_B\right)\right) = mY_A z_B + mY_B z_B \qquad (6)$$

Let ϕ be the marginal disutility caused by the pollution, assuming like Lahiri and Ono (1998) that ϕ is constant. Besides let t be the tax per unit of pollution emitted.

The welfare of the country A, W_A, will be built by the consumers' surplus of the country A, C_{SA}; the producers' surplus in the country A, $n\prod_A$; plus the tributary tax collection $t_A Z_A$, minus the total disutility times the polluting emissions in the country A, ϕZ_A, then,

$$W_A = C_{SA} + n\prod_A = t_A Z_A + \phi Z_A \qquad (7)$$

Similarly for country B, is defined by,

$$W_B = C_{SB} + m\prod_B + t_B Z_B + \phi Z_B \qquad (8)$$

If we consider the marginal costs of production of the good from the country A, s_A and the ones from the country B, s_B, we assume differences in the structures of costs between the two countries. Those costs are constants, and therefore, equivalent to the average variable costs. The prices of the good in each country are respectively p_A y p_B. In this way the benefits of the producer are given by,

$$\prod_A = \left(p_A - s_A\right)X_A + \left(p_B - s_A\right)X_B \qquad (9)$$

In other words, the marginal profit of the good, $p_A - s_A$, times the production for the local country *A*, plus the marginal profit of the homogeneous good, $p_B - s_A$, times the production of exports to the country *B*, times the number of local firms. In the same way, the producers' surplus of country *B* is given by,

$$\Pi_B = (p_B - s_B)Y_B + (p_A - s_B)Y_A \tag{10}$$

In addition the price to the homogenous good in the country *A*, is a function of the level of production of this good in the domestic industries for the local consumption, and the import level of production of this good from the foreign country, this way, by simplicity and without loss of majority we can consider the inverse function of the demand as linear and of the form,

$$p_A = \alpha_A - \beta_A D_A \qquad p_A = \alpha_A - \beta_A (nX_A + mY_A) \tag{11}$$

$$p_B = \alpha_B - \beta_B D_B \qquad p_B = \alpha_B - \beta_B (mY_B + nX_B) \tag{12}$$

Let λ be the marginal cost of abatement a unit of pollution, θ_A and θ_B, represent the quantities of pollution emitted before implementing any environmental policies. This way, the cost by each firm related with the emission of pollution is given by,

$$v_A = \lambda(\theta_A - z_A) + t_A z_A \tag{13}$$

$$v_B = \lambda(\theta_B - z_B) + t_B z_B \tag{14}$$

So that the unitary cost of production of each company is given by,

$$s_A = c_A + \lambda(\theta_A - z_A) + t_A z_A \tag{15}$$

$$s_B = c_B + \lambda(\theta_B - z_B) + t_B z_B \tag{16}$$

In these conditions z_A and z_B, represent an amount of emission of polluting agents imposed by the firms to themselves, in the understanding that they possess the technology to abate such pollution it can turn out better to reduce the amount of polluting agents that to pay a tax for the emission.

It is clear that when the tax by pollution unit is greater or just as the abatement cost the firms prefer to reduce the emission of polluting agents completely, whereas if the same tax is minor that the abatement cost, then they are continue emitting the same amount of pollution θ_A and θ_B, that is to say,

$$z_A = \begin{cases} 0 \; si \; t_A \geq \lambda \\ \theta_A \; si \; t_A < \lambda \end{cases} \qquad z_B = \begin{cases} 0 \; si \; t_B \geq \lambda \\ \theta_B \; si \; t_B < \lambda \end{cases} \tag{17}$$

And therefore,

$$s_A = \begin{cases} c_A + \lambda\theta_A \; si \; t_A \geq \lambda \\ c_A + t_A\theta_A \; si \; t_A < \lambda \end{cases} \qquad s_B = \begin{cases} c_B + \lambda\theta_B \; si \; t_B \geq \lambda \\ c_B + t_B\theta_B \; si \; t_B < \lambda \end{cases} \tag{18}$$

$$z_A = \begin{cases} 0 \; si \; t_A \geq \lambda \\ nX_A\theta_A + nX_B\theta_A \; si \; t_A < \lambda \end{cases}$$

$$z_B = \begin{cases} 0 \; si \; t_B \geq \lambda \\ mY_A\theta_B + mY_B\theta_B \; si \; t_B < \lambda \end{cases} \tag{19}$$

The calculus of the optimal tax doesn't make any sense when $t_A \geq \lambda$ and $t_B \geq \lambda$, because in this case the quantity of pollution is zero, independently from the tax amount. But when $t_A < \lambda$ and $t_B < \lambda$ all firms prefer to pay the tax and the reduction in the polluting emissions doesn´t occur, then in this case *W* does depends on *t*.

Under the previous conditions and assuming that each firm decides which proportion of the good is consumed locally, and which one is exported. Under the assumptions of Cournot-Nash, the conditions of maximization of first order are,[6]

$$\frac{d\Pi_A}{dX_A} = 0, \frac{d\Pi_A}{dX_B} = 0 \tag{20}$$

$$\frac{d\Pi_B}{dY_A} = 0, \frac{d\Pi_B}{dY_B} = 0 \tag{21}$$

6 See Appendix 1.

From which we get the solutions for variables X_A, X_B, Y_A and Y_B:[7]

$$X_A = \frac{\alpha_A - s_A + m(s_B - s_A)}{\beta_A(m+n+1)} \qquad X_B = \frac{\alpha_B - s_A + m(s_B - s_A)}{\beta_B(m+n+1)}$$

$$Y_A = \frac{\alpha_A - s_B + n(s_A - s_B)}{\beta_A(m+n+1)} \qquad Y_B = \frac{\alpha_B - s_B + n(s_A - s_B)}{\beta_B(m+n+1)} \tag{22}$$

Therefore the benefits from the firms in the country A and B in the optimal point are given by

$$\Pi_A^* = \beta_A X_A^2 + \beta_B X_B^2 \qquad \Pi_B^* = \beta_B Y_B^2 + \beta_A Y_A^2 \tag{23}$$

3. Comparative Statics

The welfare of the countries A and B is defined as the sum of the consumers' surplus plus the benefits of the firms plus the tributary tax collection minus the disutility given by pollution, this is,[8]

$$W_A = C_{SA} + n\Pi_A^* + t_A Z_A - \phi Z_A$$

$$W_B = C_{SB} + m\Pi_B^* + t_B Z_B - \phi Z_B \tag{24}$$

Differentiating W_A and W_B with respect to t_A and t_B respectively we get,[9]

$$\frac{dW_A}{dt_Z} = \frac{d(C_{SA})}{dt_A} + \frac{d(n\Pi_A^*)}{dt_A} + \frac{d(t_A Z_A)}{dt_A} - \frac{d(\phi Z_A)}{dt_A} \tag{25}$$

$$\frac{dW_A}{dt_A} = -\frac{n\theta_A(nX_A + mY_A)}{(m+n+1)} - \frac{2n\theta_A(m+1)(X_A + X_B)}{(m+n+1)}$$

[7] See Appendix 2.
[8] See Appendix 2.
[9] See Appendix 3.

$$n\theta_A\left(X_A + X_B - \frac{t_A\theta_A(m+1)(\beta_A+\beta_B)}{\beta_A\beta_B(m+n+1)}\right) - \frac{n\phi\theta_A^2(m+1)(\beta_A+\beta_B)}{\beta_A\beta_B(m+n+1)} \tag{26}$$

$$\frac{dW_B}{dt_B} = \frac{d(C_{SB})}{dt_B} + \frac{d(n\Pi_B^*)}{dt_B} + \frac{d(t_B Z_B)}{dt_B} - \frac{d(\phi Z_B)}{dt_B} \tag{27}$$

$$\frac{dW_B}{dt_B} = -\frac{m\theta_B(nX_B + mY_B)}{(m+n+1)} - \frac{2m\theta_B(m+1)(Y_A+Y_B)}{(m+n+1)}$$

$$m\theta_B\left(Y_A + Y_B - \frac{t_B\theta_B(n+1)(\beta_A+\beta_B)}{\beta_A\beta_B(m+n+1)}\right) - \frac{m\phi\theta_B^2(n+1)(\beta_A+\beta_B)}{\beta_A\beta_B(m+n+1)} \tag{28}$$

Analysing the effects of the pollution tax from the differentiated components of the welfare function, we obtain:

The profit of firms

$$d(n\Pi_A^*) = \left(\frac{2n\theta_A(m+1)(X_A+X_B)}{(m+n+1)}\right)dt_A \tag{29}$$

In this case any reduction in the pollution tax reduces the marginal costs of production of the homogeneous good, and therefore, the production in the firms is favoured, at the same time the competitiveness of the local country is increased and consequently the exports are stimulated; therefore, the benefits of the domestic firms grow. In addition, such increase in the production stimulates the employment at the same time.

The consumers' surplus

$$d(C_{SA}) = \left(-\frac{n\theta_A(nX_A + mY_A)}{(m+n+1)}\right)dt_A \tag{30}$$

Since the production costs fall for the domestic firms when the pollution tax is reduced, the prices decrease which increases the spending power of the consumers, and therefore the consumers' surplus.

The tax collection

$$d\left(t_A Z_A\right) = n\theta_A \left(X_A + X_B - \frac{t_A\theta_A\left(m+1\right)\left(\beta_A+\beta_B\right)}{\beta_A\beta_B\left(m+n+1\right)} \right) dt_A \qquad (31)$$

Clearly the tax increases the income of the government through collection of the tax from the firms and is a direct function of the levels of production of the manufacturers, although this also increases the marginal costs of the good and affects the production level negatively, reason why the combined effect is ambiguous.

Social cost for polluting

$$d\left(\phi Z_A\right) = \left(\frac{n\phi\theta_A^2\left(m+1\right)\left(\beta_A+\beta_B\right)}{\beta_A\beta_B\left(m+n+1\right)} \right) dt_A \qquad (32)$$

Evidently, reducing the tax stimulates the emissions of polluting agents to the atmosphere, thus the social cost to pollute also is increased, that is to say,

$$\frac{d\left(Z_A\right)}{dt_A} < 0 \qquad (33)$$

In the same way an increase in t_A, reduces the pollution and therefore it benefits to the country. In addition, the magnitude to such benefit depends on the size of the parameter ϕ.

Given the symmetry of the model, the same reasoning is valid for t_B, when considering the three components from the point of view of the foreign country.

Optimal tax

In order to calculate the optimal tax and to implement the conducive tax policies we make $\frac{dW_A}{dt_A} = 0$ and $\frac{dW_B}{dt_B} = 0$, finding t_A and t_B, we have,[10]

[10] See Appendix 4.

$$t_A^* = \frac{\beta_A\beta_B\left[\left(nX_B - mY_A\right) - \left(X_A + X_B\right)\left(m+1\right)\right]}{\theta_A\left(m+1\right)\left(\beta_A + \beta_B\right)} + \phi \tag{34}$$

$$t_B^* = \frac{\beta_A\beta_B\left[\left(mY_A - nX_B\right) - \left(Y_A + Y_B\right)\left(n+1\right)\right]}{\theta_B\left(n+1\right)\left(\beta_A + \beta_B\right)} + \phi \tag{35}$$

Besides we can assure that the function is concave,[11]

$$\frac{d^2W_A}{dt_A^2} = \frac{n^2\theta_A^2\left(2\beta_A\left(m+1\right)\beta_B\left(2m+1\right)\right)}{\beta_A\beta_B\left(m+n+1\right)^2} < 0 \tag{36}$$

$$\frac{d^2W_B}{dt_B^2} = \frac{m^2\theta_B^2\left(2\beta_B\left(n+1\right)\beta_A\left(2n+1\right)\right)}{\beta_A\beta_B\left(m+n+1\right)^2} < 0 \tag{37}$$

Of the expressions (34) and (35) we can observe since all the parameters are positive, the sign of t_A depends on the size of the market and of the sign and value of the parameter ϕ.

Proposition 1. In the non-cooperative equilibrium

$$t_A^* = 0 \; si \; mY_A \gg nX_B$$

$$t_B^* = 0 \; si \; nX_B \gg mY_A$$

The economic interpretation of the previous result is very intuitive. If the size of market of export of the foreign country is significantly greater than the size of market of the domestic country, then the best policy is tax rate of zero. In this case, the government favours the local firms by reducing their costs, which affects positively its benefits, increasing their competitiveness with respect to the foreign firms. At the same time it benefits the consumers who pay lower prices as a result of the reduction of marginal cost.

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11 See Appendix 5.

Although a zero tax on the emissions of polluting agents favours the increase of the pollution rate, since the firms do not have any incentive to diminish their emissions, increasing therefore the social cost to pollute. On the other hand, a zero tax prevents to the government of getting additional income by the collection of the pollution tax. Even so, in this case, the awaited benefits as much of the benefit of the firms as of consumers' surplus they surpass the adverse effects of no tax collection and a considerable increase in the social cost to pollute.

Proposition 2. In the non-cooperative equilibrium $t_A^* = 0$ and $t_B^* = 0$ if the marginal disutility to pollute ϕ is significantly elevated.

Such asseveration is obvious. The government values more the adverse effects of the pollution when the costs associated to their emission are very high, at the same time it stimulates as well to increase its tributary fundraising through the taxes. Although on the other hand, they reduce the benefits of the firms and the consumers' surplus by the increase in the marginal cost of production and consequently increases the prices to the consumer. In addition since

$$\frac{d\left(t_A^*\right)}{d\phi} > 0 \tag{38}$$

While greater it is the marginal disutility to pollute greater will be the tax determined by the government.

If we considered the case in which $m = n = 1$, that is to say, the situation of monopoly in both countries, we have:

$$t_A^* = \frac{-\beta_A\beta_B\left(2X_A + X_B + Y_A\right)}{2\theta_A\left(\beta_A + \beta_B\right)}$$

$$t_B^* = \frac{-\beta_A\beta_B\left(2Y_B + X_B + Y_A\right)}{2\theta_B\left(\beta_A + \beta_B\right)} + \phi$$

We notice that the first term of such expressions is negative, reason why the sign of t^* depends on the magnitude of ϕ, for a value of the very high disutility, a positive tax would prevail, in this case the government weights the adverse effects of the pollution against the other components of the welfare function. Whereas if such parameter is not significantly elevated then the tax would be zero, in such a way that the government cares more of the beneficial effect in the consumers' surplus and the producer's surplus due to the reduction of the marginal costs and in the price the consumers. Although it represents an increase pollution. This also is consequent with the previous proposition.

On the other hand, the function W not necessarily is continuous with respect to t.

For the way that s_A y s_B are define, the only possible point of discontinuity is $t = \lambda$. Analyzing the likely discontinuity of W in $t = \lambda$ by calculating unilateral limits and using (17), (18) y (19) we have that,

$$lim_{t\to\lambda} + W_A = C_{SA} + n\prod_A^* + t_A Z_A - \phi Z_A \tag{39}$$

$$lim_{t\to\lambda} + W_A = C_{SA} + n\prod_A^* \tag{40}$$

$$lim_{t\to\lambda} - W_A = C_{SA} + n\prod_A^* + t_A Z_A - \phi Z_A \tag{41}$$

$$lim_{t\to\lambda} - W_A = C_{SA} + n\prod_A^* + \lambda\left(nX_A\theta_A + nX_B\theta_A\right) - \phi\left(nX_A\theta_A + nX_B\theta_A\right) \tag{42}$$

$$lim_{t\to\lambda} - W_A = C_{SA} + n\prod_A^* + \left(\lambda - \phi\right)\left(nX_A\theta_A + nX_B\theta_A\right) \tag{43}$$

Therefore from (40) and (43) we have,

$$lim_{t\to\lambda} + W_A - lim_{t\to\lambda} - W_A = \left(\lambda - \phi\right)\left(nX_A\theta_A + nX_B\theta_A\right)$$

Where we concluded that,

$$lim_{t\to\lambda} + W_A - lim_{t\to\lambda} - W_A > 0 \ si\ \phi > \lambda \tag{44}$$

$$lim_{t\to\lambda} + W_A - lim_{t\to\lambda} - W_A = 0 \ si\ \phi = \lambda \tag{45}$$

$$lim_{t\to\lambda} + W_A - lim_{t\to\lambda} - W_A < 0 \ si\ \phi < \lambda \tag{46}$$

For a similar reasoning we get,

$$lim_{t\to\lambda} + W_B - lim_{t\to\lambda} - W_B = (\lambda - \phi)(mY_A\theta_B + mY_B\theta_B)$$

Where we concluded that,

$$lim_{t\to\lambda} + W_B - lim_{t\to\lambda} - W_B > 0 \ si \ \phi > \lambda \tag{47}$$

$$lim_{t\to\lambda} + W_B - lim_{t\to\lambda} - W_B = 0 \ si \ \phi = \lambda \tag{48}$$

$$lim_{t\to\lambda} + W_B - lim_{t\to\lambda} - W_B < 0 \ si \ \phi < \lambda \tag{49}$$

Which we can summarize on the next proposition.

Proposition 3. If $\phi \geq \lambda$ then the tax $t_A^* \geq \lambda$ therefore there´s no polluting agents emission. And if $\phi < \lambda$ then the tax $t_A^* < \lambda$ therefore there´s no reduccion on the pollutant emission.

Intuitively if the disutility to pollute is very high compared to the abatement cost the benefit of reducing the emission of pollutants is imposed on other components of the welfare function, implementing the tax is higher than the abatement cost that´s why firms prefer not to emit pollutants at all.

While if the marginal disutility is not significantly high compared with the abatement cost, the optimal tax is strictly less than the abatement cost and in this case, firms choose not to reduce their emissions.

4. Conclusions

One of the most used instruments for environmental policy to regulate the emission of pollutants into the environment which would not depend on the willingness of firms, is the imposition of governments of a tax per unit of pollution emitted, *i.e.*, the government put a price on pollution, and firms pay the government in proportion to the amount of emissions they generate, as theoretically correct the market failure that causes the production of goods through the pollution, as the government to intervene by imposing a pollution tax must be costly and so the firms weighs on purely economic criteria the viability to pollute less.

In this project we develop an environmental policy model by taxes to the emission of pollutants under conditions of oligopolistic competition.

We consider trade between two small countries of similar size, assuming that reciprocal dumping exists. So, firms dedicate most of their production to the local consumption and the other part to export. We assume that firms pollute as part of the production process, but at the same time they must have technology to lower pollutant emissions. Under these circumstances we calculate the optimal pollution tax. Also from the optimal tax derived a series of strategic policies that are related with the costs structure of the firms and in particular the abatement cost and disutility from pollution. These environmental policies directly affect the welfare function of the countries and their components (consumers' surplus, firms profits and social cost for polluting).

The magnitude of the optmimal tax depends mainly on the export market size of countries and the size of the marginal disutility of pollution. In the first case, if the size of the country's export market is larger than the local market, the government burden on domestic firms a zero tax rate, which enhances the competitiveness of such firms to reduce the marginal costs of production, inducing a positive effect on consumers' surplus and in firms benefit. Although the zero-tax results in an increase in pollution and social costs that these involve. On the other hand, if the marginal inutility for pollution is very high, then the government values more the adverse effects of pollution, which at the same time increases tax collection by means of the pollution tax, although this sacrifices in some way the benefit to companies and consumer well-being. The above result is also true in the case of monopolies in both countries.

Finally, if we compare the marginal disutility with the abatement cost we conclude that if the first is greater than the second, then the optimal tax must be greater than the abatement cost and in this case the firms decide not to pollute at all, because clearly it is cheaper to cover the cost of not polluting than pay an expensive tax.

In the opposite case, when the marginal disutility of pollution is less than the abatement cost, then the optimal tax should be less than the abatement cost, in which case firms choose not to reduce the least emission of pollutants, because bringing down the abatement cost the pollution is clearly much more expensive than paying the taxes on the emission of pollutants.

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Appendix

1. First order conditions

The profits of the firms in the country *A* and *B* are given by,

$$\Pi_A = (p_A - s_A)X_A + (p_B - s_A)X_B$$

$$\Pi_B = (p_B - s_B)Y_B + (p_A - s_B)Y_A$$

We also know that,

$$p_A = \alpha_A - \beta_A D_A$$

$$p_A = \alpha_A - \beta_A(nX_A + mY_A)$$

$$p_B = \alpha_B - \beta_B D_B$$

$$p_B = \alpha_B - \beta_B(mY_B + nX_B)$$

Replacing p_A, p_B, s_A y s_B, en Π_A y Π_B we have:

$$\Pi_A = \Big(\big(\alpha_A - \beta_A(nX_A + mY_A)\big) - s_A\Big)X_A + \Big(\big(\alpha_B - \beta_B(mY_B + nX_B)\big) - s_A\Big)X_B \tag{50}$$

$$\Pi_B = \Big(\big(\alpha_B - \beta_B(mY_B + nX_B)\big) - s_B\Big)Y_B + \Big(\big(\alpha_A - \beta_A(nX_A + mY_A)\big) - s_B\Big)Y_A \tag{51}$$

Differentiating with respect to X_A, X_B, Y_A and Y_B to obtain the values of those variables that maximize the profits of the firms in both countries we have,

$$\frac{d\Pi_A}{dX_A} = \alpha_A - s_A - 2X_A\beta_A + \beta_A X_i - m\beta_A Y_A - n\beta_A X_i$$

$$\frac{d\Pi_A}{dX_A} = \alpha_A - s_A - 2\beta_A X_A + \beta_A X_A - m\beta_A Y_A - n\beta_A X_A$$

$$\frac{d\Pi_A}{dX_A} = \alpha_A - s_A - \beta_A X_A - \beta_A (mY_A + nX_A) = 0 \tag{52}$$

But, $p_A = \alpha_A - \beta_A (nX_A + mY_A)$, then

$$\beta_A X_A = p_A - s_A \tag{53}$$

Similarly, performing the same calculations for, $\frac{d\Pi_A}{dX_B}$, $\frac{d\Pi_B}{dY_A}$ y $\frac{d\Pi_B}{dY_B}$ results

$$\frac{d\Pi_B}{dX_B} = \alpha_B - s_A - \beta_B X_B - \beta_B (mY_B + nX_B) = 0 \tag{54}$$

$$\beta_B X_B = p_B - s_A \tag{55}$$

$$\frac{d\Pi_B}{dY_B} = \alpha_A - s_B - \beta_A Y_A - \beta_A (mY_A + nX_A) = 0 \tag{56}$$

$$\beta_A Y_A = p_A - s_B \tag{57}$$

$$\frac{d\Pi_B}{dY_B} = \alpha_B - s_B - \beta_B Y_B - \beta_B (mY_B + nX_B) = 0 \tag{58}$$

$$\beta_B X_B = p_B - s_B \tag{59}$$

2. Closed solutions for the variables X_A, X_B, Y_A and Y_B

Solving the system of simultaneous equations given by the reaction curves, expressed by the equations (52), (54), (56) y (58), that express the Cournot equilibrium, we find the optimal choice of output level X_A, X_B, Y_A and Y_B for the firms of the country *A* and the country *B*.

$$X_A = \frac{\alpha_A - s_A + m(s_B - s_A)}{\beta_A(m+n+1)} \tag{60}$$

$$X_B = \frac{\alpha_B - s_A + m(s_B - s_A)}{\beta_B(m+n+1)} \tag{61}$$

$$Y_A = \frac{\alpha_A - s_B + n(s_A - s_B)}{\beta_A(m+n+1)} \tag{62}$$

$$Y_B = \frac{\alpha_B - s_B + n(s_A - s_B)}{\beta_B(m+n+1)} \tag{63}$$

Replacing (60), (61), (62), (63) in (50) and (51), we get the company's benefits in the country *A* and *B* at the optimal point,

$$\prod\nolimits_A^* = \beta_A X_A^2 + \beta_B X_B^2 \tag{64}$$

$$\prod\nolimits_B^* = \beta_B Y_B^2 + \beta_A Y_A^2 \tag{65}$$

3. Total differentiation of the welfare function

We will differentiate the functions of welfare for the country *A* and the country *B*

$$W_A = C_{SA} + n\prod\nolimits_A^* + t_A Z_A - \phi Z_A \tag{66}$$

$$W_B = C_{SB} + m\prod\nolimits_B^* + t_B Z_B - \phi Z_B \tag{67}$$

But before X_A, X_B, Y_A and Y_B respect to taxes in the respective countries (will be use in the next calculations). Replacing (15) and (16) in (60), (61), (62), (63) and differentiating with respect to t_A and t_B, we get,

$$\frac{dX_A}{dt_A} = \frac{-\theta_A(m+1)}{\beta_A(m+n+1)} \tag{68}$$

$$\frac{dX_A}{dt_B} = \frac{m\theta_B}{\beta_A(m+n+1)} \tag{69}$$

$$\frac{dY_A}{dt_A} = \frac{n\theta_A}{\beta_A(m+n+1)} \tag{70}$$

$$\frac{dY_A}{dt_B} = \frac{-\theta_B(n+1)}{\beta_A(m+n+1)} \tag{71}$$

$$\frac{dX_B}{dt_A} = \frac{-\theta_A(n+1)}{\beta_B(m+n+1)} \tag{72}$$

$$\frac{dX_B}{dt_B} = \frac{m\theta_B}{\beta_B(m+n+1)} \tag{73}$$

$$\frac{dY_B}{dt_A} = \frac{n\theta_A}{\beta_B(m+n+1)} \tag{74}$$

$$\frac{dY_B}{dt_B} = \frac{-\theta_B(n+1)}{\beta_B(m+n+1)} \tag{75}$$

Consumers' surplus

To derive the first term of (66) and (67), given the demand of the countries A and B, consumers' surplus is given by,

$$C_{SA} = \frac{\beta_A D_A^2}{2} = \frac{\beta_A(nX_A + mY_A)^2}{2} \tag{76}$$

$$C_{SB} = \frac{\beta_B D_B^2}{2} = \frac{\beta_B(nX_B + mY_B)^2}{2} \tag{77}$$

Differentiating the above equations respect to t_A and t_B, and using (68), (70), (73) y (75) we get,

$$\frac{dC_{SA}}{dt_A} = \frac{-n\theta_A(nX_A + mY_A)}{(m+n+1)} \tag{78}$$

$$\frac{dC_{SB}}{dt_B} = \frac{-m\theta_B(nX_B + mY_B)}{(m+n+1)} \tag{79}$$

The benefit of the firms

In order to derive the second term of (66) and (67), given the benefits of the firms in the countries A and B (64) and (65); and using again (68), (71), (72) and (75) we get,

$$\frac{d(n\Pi_A^*)}{dt_A} = n\frac{d(\Pi_A^*)}{dt_A} = n\frac{d(\beta_A X_A^2 + \beta_B X_B^2)}{dt_A} = -\frac{2\theta_A n(m+1)(X_A + X_B)}{m+n+1} \tag{80}$$

$$\frac{d(n\Pi_B^*)}{dt_B} = m\frac{d(\Pi_B^*)}{dt_B} = m\frac{d(\beta_B Y_B^2 + \beta_A Y_A^2)}{dt_B} = -\frac{2\theta_B m(n+1)(Y_A + Y_B)}{m+n+1} \tag{81}$$

The tributary component

Differentiating the fourth term from (66) and (67), and starting off of the total emission of polluting agents in the countries A and B by the disutility to pollute ϕ.

$$t_A Z_A = t_A(nX_A\theta_A + nX_B\theta_A)$$

$$t_B Z_B = t_B(mY_A\theta_B + mY_B\theta_B)$$

And using (68), (71), (72) and (75) we have the following thing,

$$\frac{d(t_A Z_A)}{dt_A} = \frac{d(t_A((nX_A\theta_A + nX_B\theta_A)))}{dt_A} = n\theta_A\left(X_A + X_B - \frac{t_A\theta_A(m+1)(\beta_A + \beta_B)}{\beta_A\beta_B(m+n+1)}\right) \tag{82}$$

$$\frac{d\left(t_B Z_B\right)}{dt_B}=\frac{d\left(t_B\left(mY_A\theta_B+mY_B\theta_B\right)\right)}{dt_B}=m\theta_B\left(Y_A+Y_B-\frac{t_B\theta_B\left(n+1\right)\left(\beta_A+\beta_B\right)}{\beta_A\beta_B\left(m+n+1\right)}\right) \tag{83}$$

Social cost to pollute

Differentiating the third term from (66) and (67), and starting off to the total emission of polluting agents in the countries A and B by the disutility to pollute ϕ.

$$\phi Z_A=\phi\left(nX_A\theta_A+nX_B\theta_A\right)$$

$$\phi Z_B=\phi\left(mY_A\theta_B+mY_B\theta_B\right)$$

thus, using again (68), (71), (72) and (75) we get the following,

$$\frac{d\left(\phi Z_A\right)}{dt_A}=\phi\frac{d\left(Z_A\right)}{dt_A}=\phi\frac{d\left(nX_A\theta_A+nX_B\theta_A\right)}{dt_A}=-\frac{n\phi\theta_A^2\left(m+1\right)\left(\beta_A+\beta_B\right)}{\beta_A\beta_B\left(m+n+1\right)} \tag{84}$$

$$\frac{d\left(\phi Z_B\right)}{dt_B}=\phi\frac{d\left(Z_B\right)}{dt_B}=\phi\frac{d\left(mY_A\theta_B+mY_B\theta_B\right)}{dt_B}=-\frac{m\phi\theta_B^2\left(n+1\right)\left(\beta_A+\beta_B\right)}{\beta_A\beta_B\left(m+n+1\right)} \tag{85}$$

Therefore, $\frac{dW_A}{dt_A}$ is

$$\frac{dW_A}{dt_A}=\frac{d\left(C_{SA}\right)}{dt_A}+\frac{d\left(n\prod_A^*\right)}{dt_A}+\frac{d\left(t_A Z_A\right)}{dt_A}-\frac{d\left(\phi Z_A\right)}{dt_A}$$

$$\frac{dW_A}{dt_A}=-\frac{n\theta_A\left(nX_A+mY_A\right)}{\left(m+n+1\right)}-\frac{2n\theta_A\left(m+1\right)\left(X_A+X_B\right)}{\left(m+n+1\right)}$$

$$+n\theta_A\left(X_A+X_B-\frac{t_A\theta_A\left(m+1\right)\left(\beta_A+\beta_B\right)}{\beta_A\beta_B\left(m+n+1\right)}\right)-\left(\frac{n\phi\theta_A^2\left(m+1\right)\left(\beta_A+\beta_B\right)}{\beta_A\beta_B\left(m+n+1\right)}\right) \tag{86}$$

And similarly $\frac{dW_B}{dt_B}$ we get ,

$$\frac{dW_B}{dt_B}=\frac{d(C_{SB})}{dt_B}+\frac{d(n\Pi_B^*)}{dt_B}+\frac{d(t_B Z_B)}{dt_B}-\frac{d(\phi Z_B)}{dt_B}$$

$$\frac{dW_B}{dt_B}=-\frac{m\theta_B(nX_B+mY_B)}{(m+n+1)}-\frac{2n\theta_B(m+1)(Y_A+Y_B)}{(m+n+1)}$$

$$+m\theta_B\left(Y_A+Y_B-\frac{t_B\theta_B(n+1)(\beta_A+\beta_B)}{\beta_A\beta_B(m+n+1)}\right)-\left(\frac{m\phi\theta_B^2(n+1)(\beta_A+\beta_B)}{\beta_A\beta_B(m+n+1)}\right) \quad (87)$$

4. Optimal tax of pollution

If we do $\frac{dW_A}{dt_A}$ y $\frac{dW_B}{dt_B}$ in order to find the optimal tax in (86) and in (87), clearing t_A y t_B, we get,

$$t_A^*=\frac{\beta_A\beta_B\left[(nX_B-mY_A)-(X_A+X_B)(m+1)\right]}{\theta_A(m+1)(\beta_A+\beta_B)}+\phi \quad (88)$$

$$t_B^*=\frac{\beta_A\beta_B\left[(mY_A-nX_B)-(Y_A+Y_B)(n+1)\right]}{\theta_A(m+1)(\beta_A+\beta_B)}+\phi \quad (89)$$

5. Concavity of the welfare function

Obtaining the second derivative from the welfare function with respect to the tax to determine the conditions of concavity we have,

$$\frac{d^2W_A}{dt_A^2}=-\frac{n^2\theta_A^2(2\beta_A(m+1)+\beta_B(2m+1))}{\beta_A\beta_B(m+n+1)^2}<0 \quad (90)$$

$$\frac{d^2W_B}{dt_B^2}=-\frac{m^2\theta_B^2(2\beta_B(n+1)+\beta_A(2n+1))}{\beta_A\beta_B(m+n+1)^2}<0 \quad (91)$$

Then, W is concave.