https://doi.org/10.29201/peipn.v9i17.302
PANORAMA ECONÓMICO, Volumen V, No 9,
julio-diciembre, 2009, pp 7-54

WHO WANTS TO BE A GENIUS?

Dmitri Fujii*

RESUMEN

En este artículo desarrollamos un modelo teórico para una economía dual, en la que los dos tipos de individuos (empresas) escogen entre convertirse en inventores y copiar las ideas desarrolladas por otros. Se analiza un caso específico: algunos individuos son genios y todos los genios se vuelven inventores mientras los individuos ordinarios se convierten en piratas. El modelo está basado en un trabajo previo de Grossman (2005), pero incluye dos nuevas aspectos, una etapa previa en la que los individuos toman la decisión de convertirse en genios (mediante un pago fijo) y un análisis de bienestar para la economía. La principal conclusión del artículo es que, a pesar de que una mayor proporción de genios en la economía representa un mayor bienestar, las políticas implementadas por las autoridades (en el caso de los derechos de propiedad, entre otros instrumentos) para crear incentivos que incremen esta proporción, depende de las condiciones iniciales de la economía (en términos de la proporción de genios).

Palabras clave: Economía dual, agentes económicos, condiciones iniciales, derechos de propiedad, bienestar
Clasificación JEL: O31, O34

---

* Profesor Investigador del Colegio Mexiquense, A. C. Correo-e: <dmitri.fujii@gmail.com>.

ABSTRACT

In this paper we present a theoretical model of a dual economy, in which the two groups of individuals (firms) choose between becoming inventors and copying other individuals` ideas. A particular case, in which some individuals are geniuses and all geniuses become inventors while all ordinary people become pirates, is analyzed. The model is based on Grossman (2005), but considers two main changes: a previous stage is revised (in which there is a fixed cost of becoming a genius) and a wealth analysis for the economy is presented. The main conclusion of the paper is that even when a higher proportion of geniuses represent a higher welfare for the society, the policy implemented by the authorities (in terms of intellectual property rights, among other instruments) to create incentives for such an increase depends on the initial conditions of the economy (in terms of the proportion of geniuses itself).

Palabras clave: Dual economy, economic agents, initial conditions, property rights, welfare

JEL classification: O31, O34

1. INTRODUCTION

Recently, many authors have argued that within the unbalanced development of Mexican manufactures, two types of sectors have emerged: the *successful* and *unsuccessful* ones (Arjona and Unger, 1996; Brown and Dominguez, 1999; Cimoli, 2000). Also, there are several studies that propose a specific distinction to identify these successful and unsuccessful sectors (Casar, 1993; Dussel, 1994; Fujii, 2004).

The empirical evidence of a dual structure for the manufacturing industry can be formalized by a theoretical model. This is the aim of this paper. A producer-thief type of economy framework is used to analyze the dynamics that arise when a firm has the choice to invest in knowledge and become an innovator or remain as a copier of others' ideas.

In particular, our model is based on a recent study by Grossman (2005) for an economy with inventors and pirates, where the two groups share the value of inventions obtained by the inventors. Extending this model, a previous stage is analyzed: an individual (firm) faces a decision of investing and becoming an inventor or remaining as a pirate.

The results of the theoretical model suggest that a policy of intellectual property rights (IPRs) protection is not always effective, but depends on the initial conditions of the industry (in terms of the existing inventors). Finally, the implications for society's welfare in this framework are presented.

2. LITERATURE REVIEW

The roles of technology and innovation protection are central aspects in the inventor-pirate dual structure literature. This line of research not only focuses on the innovative side of the economy, but also on a second type of individuals (or firms): imitative or copying ones. The interaction between the inventors (or producers) and this second type of individuals (pirates), the IPRs enforcement and society's welfare are the main concerns of this approach in the economic literature.

The pioneering work of Becker (1968) initiated an economic approach to the analysis of crime on the society. The implicit assumption behind the logic of this approach is that, under some circumstances, being a criminal could be an economically-rational activity. In a subsequent study, Becker and Stigler (1974) suggest that it would be useful to extend private enforcement mechanisms to situations where the law is enforced publicly, as public enforcement has inefficiencies.

Later studies, such as Landes and Posner (1975) and Friedman (1984) focus on the debate initiated by Becker works on the inefficiencies of private enforcement institutions. These studies develop economic modeling for private and public enforcement institutions and the socially optimal amount of enforcement.

Neher (1978) conducts an economic analysis of muggery (wealth transfers from muggees to muggers). The author develops a dynamic model that includes cost and benefits of muggery in a society of free entry (or uncontrolled) for muggers. Later on, he moves towards a model of controlled muggery, based on the assumption that a perfectly, competitive muggery environment will reduce muggers` profits to zero.

Another study that formalizes the producer-thief relationship is Usher (1987). The paper analyzes the welfare cost of theft and formalizes several possible ways of losing efficiency in this context: loss of labor of the thief, loss of labor of the victim, destruction of product and underproduction of stealable good.

Based on the pioneering works described before, there have been several studies on the inventors-pirates' dual structure in recent years. The common aspects of these studies are the individual's choice of becoming an inventor or a pirate and the importance of protection of inventors' ideas under these circumstances.

Grossman and Kim (1995) present a general equilibrium model, in which two individuals decide the allocation of their resources among productive and appropriative activities, following the predator-pray relationship formalized in Neher (1978) or Usher (1987). The results reveal that a minimum defensive allocation of resources is needed and that with high protection of property the cost of appropriation activities is higher.

Grossman and Kim (1996) extend their previous model to focus on equilibrium with pure predation (where an agent decides to allocate all his initial endowments to predatory activities). According to their model, this type of equilibrium is possible if the initial endowment of the predator is small and if the technology is such that the weapons are neither too effective, neither too ineffective against fortifications.

Grossman (1998) develops another predator-prey model with a continuum of people that are potential predators or preys, depending on their allocation of initial endowments. In this model, the decision of resources' allocation can be taken either individually or collectively, based on law enforcement. This decision depends on the consumption that each alternative (being a producer or a predator) yields. The results suggest that given the optimal amount of defensive resources, a collective decision of resources' allocation yields a larger consumption than that of an individual decision, and that the social cost of predation is smaller when the decision is taken collectively.[1]

Grossman and Kim (2002; 2003) develop similar models to the previous ones: individuals split their resources between productive and appropriative activities. In their first model, the distribution of consumption (following the Rawlsian criterion of maximizing the consumption of the poorest individual) is considered. Meanwhile, their second model focuses more on the decision of egalitarian or elitist type of educational policy, in a model of well-endowed (with human capital) and poorly endowed individuals.

3. THE INVENTOR-PIRATE FRAMEWORK

The most recent analysis of the inventor-pirate dual structure is Grossman (2005). In his model, each potentially creative person chooses either to become an inventor or a pirate, depending on which one yields more wealth. As a result of this decision, there is a proportion of the society that is pirates (r), while the rest are inventors ($1-r$), with a ratio of:

$$R = \frac{r}{1-r} \tag{1}$$

Each inventor chooses the amount of creative activity, dedicating a proportion of their time to creating new ideas ($1-g$) and the rest of the time to guarding those ideas (g), with a ratio of:

[1] Cozzi (2001) presents a similar decision-based model. In his framework, similarly endowed R&D engineers allocate their efforts in creation of new ideas or spying the ideas developed by fellow engineers. The results of Cozzi's model reveal that the larger the skilled population, the higher are the incentives to spy in this environment.

$$G = \frac{g}{1-g} \tag{2}$$

There is a (exogenous) saleable value of ideas, Ω, which has a real value of $(1-g)\,\Omega$ that is shared by inventors and pirates:

$$(1-g)\Omega = \frac{\Omega}{1+G} \tag{3}$$

This sharing means that the inventor retains a proportion (p) of the real (discounting the guarding time) saleable value of the ideas created, while the rest of it, $(1-p)$, goes to pirates. This proportion depends negatively on the effectiveness of pirating (θ), which, in turn, depends on the IPRs protection (following an rise in IPRs protection, it becomes more difficult to pirate, *i.e.* more protection = lower θ) and the proportion of pirates (R); and positively on the time allocated to guarding (G). Formally, p is assumed to be:

$$p = \frac{1}{1+\frac{\theta R}{G}} \tag{4}$$

The decision to become an inventor or a pirate depends solely on which yields more wealth. The wealth of an inventor is:

$$C = p(1-g)\Omega \ , \tag{5}$$

Or

$$C = \frac{p\Omega}{1+G} \tag{5'}$$

While a pirate receives a wealth of:

$$D = \frac{1-r}{r}(1-p)(1-g)\Omega, \tag{6}$$

Or

$$D = \frac{1-p}{R}\frac{\Omega}{1+G} \tag{6'}$$

Comparing the potential wealth under each case (pirate or inventor), and taking the proportion of pirates (R) as given, Grossman solves for the guarding time that maximizes inventor's wealth (G^*), which is:

$$G^* = \sqrt{\theta R} \tag{7}$$

Then, comparing the inventor's wealth and the pirate's wealth, taking G as given, he solves for the optimal proportion of pirates in the society (R^*), which is:

$$R^* = \theta \tag{8}$$

Therefore, equation (4) becomes:

$$p = \frac{1}{1+\theta} \tag{9}$$

Hence, the main conclusions of this model are that when the environment for pirating is better (*i.e.* a higher θ), the proportion of inventors is lower and more time is allocated to guarding.[2]

---

[2] The results of this model suggest that there is a loss of efficiency (in the form of underproduction of the stealable good, the ideas created) first stressed by Usher (1987). In general, Grossman formalizes Usher's analysis in an environment in which an individual chooses between becoming an inventor or a pirate, given the potential wealth of each choice.

The model is then extended to consider two types of potential inventors: ordinary people and geniuses. It is assumed that a proportion of the society is geniuses (e), while the rest are ordinary people ($1-e$), with a ratio of:

$$E = \frac{e}{1-e} \tag{10}$$

To distinguish between a genius and an ordinary person, it is assumed that the value of ideas created by a genius (Ω_e) is higher than the one created by an ordinary person (Ω_0). Thereafter, society is divided in three types of individuals: ordinary inventors (V_0), geniuses that are inventors (V_e) and pirates:[3]

$$V_e + V_o + r = 1 \tag{11}$$

Given this modified context, a genius (as well as an ordinary person) chooses to become an inventor or a pirate. The decision depends on the wealth that each alternative yields, which Grossman shows to be:

$$C_e = \frac{p\Omega_e}{1+G} \tag{12}$$

$$C_o = \frac{p\Omega_o}{1+G} \tag{13}$$

$$D = \frac{1-p}{r}(1-g)(V_e\Omega_e + V_o\Omega_o) \tag{14}$$

[3] Grossman assumes that the ability of pirating is exogenous and fixed, even with the presence of geniuses in the economy. This means that if a genius chooses to become a pirate, his ability to pirate will be the same as that of an ordinary pirate.

$$D = \frac{(1-p)}{R\;1+R}\,\frac{(V_e\Omega_e + V_o\Omega_o)}{1+G} \tag{14'}$$

Where C_e, C_0 and D represent the wealth of a genius-inventor, the wealth of an ordinary inventor and the wealth of a pirate, respectively.

As the equations for wealth of inventors in the extended model (12, 13) are, in fact, similar to that of inventors wealth without geniuses in the economy (equation 5), inventors solve for the optimal guarding time (G^*) in the same way as before, taking their wealth and the proportion of pirates (R) as given. In this case, the optimal guarding time will have the same value as before:

$$G^* = \sqrt{\theta R} \tag{7}$$

However, the optimal proportion of pirates (R^*) will depend on the relationship between C_e, C_0 and D, as the choice of being a pirate or an inventor has to be considered for two types of individuals now: ordinary ones and geniuses. In this case, the author describes four possible equilibria:

1. $C_e > D > C_0$ (all geniuses are inventors; all ordinary people are pirates),
2. $C_e > C_0 > D$ (everybody is an inventor),
3. $C_e > C_0 = D$ (ordinary people are indifferent between inventing and pirating),
4. $C_e = D > C_0$ (geniuses are indifferent between inventing and pirating).

Grossman's subsequent analysis focuses on the first case: the separating equilibrium. For this equilibrium several results are derived, as presented below.

Firstly, as the proportion of pirates (r) is equivalent to that of ordinary people ($1\text{-}e$) and the proportion of geniuses (e) is equivalent to that of inventors ($1\text{-}r$), we can combine equations (1) and (10) to obtain the following relationship between the proportion of geniuses and pirates:[4]

$$R = \frac{1}{E} \tag{15}$$

Substituting (15) into (7):

$$G = \sqrt{\frac{\theta}{E}} \tag{16}$$

And substituting both R and G in (4), the proportion of the value of ideas created (p) that the inventor retains (in the separating equilibrium) is given by:

$$p = \frac{1}{1+\sqrt{\theta/E}} \tag{17}$$

As there are no ordinary inventors, their benefits, given by equation (13), are zero and only the value of ideas created by a genius (Ω_e) is relevant for this equilibrium. Therefore, the benefits of an (genius) inventor will be given by:

$$C_e = \frac{p\Omega_e}{1+G} \tag{18}$$

[4] Note that (15) applies only for the separating equilibrium, as the proportion of pirates (r) is the same as the proportion of ordinary people ($1\text{-}e$). Similarly, equations (16) and (17) apply only for the separating equilibrium.

Or, substituting (16) and (17) into (18):

$$C_e = \frac{\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2} \tag{18'}$$

Meanwhile, the benefits of a (ordinary) pirate are given by:

$$D = \frac{(1-p)}{R}\frac{\Omega_e}{1+G} \tag{19}$$

Or, substituting (15), (16) and (17) into (19):

$$D = \frac{\sqrt{\theta E}\,\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2} \tag{19'}$$

The results (optimal G^* and R^*) of the separating equilibrium show that, in this case, both the proportion of pirates (R) and the guarding time (G) are higher with the presence of geniuses in the economy, than with ordinary people only. Grossman concludes that for this equilibrium a larger fraction of people chooses to be a pirate, and inventors allocate more time to guarding ideas than in the simple version of the model.

Although the results of the extended Grossman's model are intriguing, they are not very useful, unless some modeling of policy instruments to control the number of geniuses in the economy or the way geniuses are formed is included. These aspects are crucial for the analysis conducted in the following section.

4. THE PREVIOUS STAGE: WHO WANTS TO BE A GENIUS?

In the framework described above, the proportion of geniuses ordinary people is exogenously given. The main interest of this paper is to see how this proportion is determined. This analysis will be useful in understanding how a dual structure is created within an economy, how these sectors can be characterized, the dynamics of this division, and the effects of policy instruments in this environment.

The present analysis focuses on the separating equilibrium only, as it is the case most relevant to an observation of a dual economy. In this case, those individuals that choose to be a genius in the first stage will be inventors in the second one, while those that prefer to remain ordinary will become pirates in the second stage. In other words, once you have decided not to pay the investment cost, you remain as a pirate. Hence, in our model there will be some geniuses and some pirates in the second stage, which implies a "splitting" economy. At the first stage, some people will invest to become geniuses while some will remain ordinary and, at the second stage, all geniuses are inventors and all ordinary people are pirates. We refer to this as a "splitting then separating" type of economy.

To determine the choice of becoming a genius or remaining as an ordinary person, let K be the fixed cost of investment in education or training. Consider the benefits of becoming a genius as the difference between geniuses' payoffs and the investment cost, and the benefits of an ordinary person as those of a pirate in the second stage.

In this case, there will be three possible situations, only one of which is compatible with Grossman's separating equilibrium: the one when an individual is indifferent between investing in becoming a genius and remaining an ordinary pirate.[5] This situation is given by the following condition for a "splitting" equilibrium:

[5] The other two situations are: $C_e - K > D$ (everybody prefers to become a genius) and $C_e - K < D$ (everybody prefers to remain ordinary and be a pirate). For the first one, $e = 1$ and for the second, $e = 0$. As these cases are extreme, we will focus on the intermediate one, for which the individual is indifferent between becoming a genius or remaining as an ordinary pirate, in which case $0 < e < 1$.

$$C_e - K = D \tag{20}$$

In order to combine the results of a separating equilibrium with equation (20), we substitute the values of the inventors' and pirates' wealth, given by (18) and (19) respectively, into (20):

$$\frac{p\Omega_e}{1+G} - K = \frac{(1-p)}{R}\frac{\Omega_e}{1+G} \tag{21}$$

Or, using (18') and (19'):

$$\frac{\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2} - K = \frac{\sqrt{\theta E}\,\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2} \tag{22}$$

Which, in turn, can be reduced to:

$$\frac{\left(1-\sqrt{\theta E}\right)}{\left(1+\sqrt{\theta/E}\right)^2} = \frac{K}{\Omega_e} \tag{23}$$

It can be seen from equation (23) that a necessary condition for this expression to hold when the value of ideas and the investment cost are positive (Ω_e, $K > 0$) is $\sqrt{\theta E} < 1$, *i.e.* the squared root of the product of the effectiveness of pirating with the proportion of geniuses should be less than one. Additionally, the value of ideas should be higher than the investment cost, and both should be positive for a positive E^* (see Appendix).

We will specify this as a formal assumption in what follows:[6]

$$\sqrt{\theta E} < 1 \qquad \text{And} \qquad \Omega_e > K > 0 \tag{A1}$$

The splitting economy, represented by equation (23), can be solved for the proportion of geniuses in equilibrium (E^*). (23) Can be rewritten as a cubic equation (see Appendix):

$$\left(\frac{\Omega_e}{\theta^{1/2}K}\right)E^{3/2} - \left(\frac{\Omega_e - K}{\theta K}\right)E + \left(\frac{2}{\theta^{1/2}}\right)E^{1/2} + 1 = 0 \tag{24}$$

There are three solutions to (24): one real and two imaginary. The (real) solution of (24) is positive for the parameter range described by (A1). This implies that there does exist an interior solution for the splitting equilibrium (*i.e.* there is a positive proportion of geniuses in equilibrium, E^*).[7]

Once it has been proved that there is a positive (real) solution for E^*, the dynamics of geniuses can be described from the splitting equilibrium equations. For instance, from equation (23), it is possible to determine how the proportion of geniuses will vary when there is an increase in the investment cost (K), the effectiveness of pirating (θ), or the value of ideas created (Ω_e).

---

[6] If $\sqrt{\theta E} > 1$, then for any $\Omega_e > K > 0$ we have an equilibrium where all firms choose to remain as pirates.

[7] The derivation of the solution for E^* can be seen in the Appendix. As the real solution of the cubic equation (24) is not manageable (or easy to read), it is presented only in the Appendix of the present chapter. However, the main conclusion of this exercise is that an interior solution for the splitting economy does exist, and that it is possible to calculate a positive proportion of geniuses (E^*) in equilibrium.

As there is an interior solution for (23), we can use the implicit function theorem (IFT),[8] to find the impact of cost, protection, and value of ideas, on the proportion of geniuses: dE/dK, $dE/d\theta$ and $dE/d\Omega_e$. Using equation (23), if we define $f(\cdot)$ as follows:

$$f(E,\theta,\Omega_e,K)=\left(1-\sqrt{\theta E}\right)\Omega_e-K\left(1+\sqrt{\theta/E}\right)^2=0 \tag{25}$$

$$f(\bullet)=\left(1-\theta^{1/2}E^{1/2}\right)\Omega_e-K\left(1+\theta^{1/2}E^{-1/2}\right)^2=0 \tag{25'}$$

Then, the impacts discussed above are given by:

$$\frac{dE}{dK}=-\frac{\partial f/\partial K}{\partial f/\partial E} \tag{26}$$

$$\frac{dE}{d\theta}=-\frac{\partial f/\partial \theta}{\partial f/\partial E} \tag{27}$$

$$\frac{dE}{d\Omega_e}=-\frac{\partial f/\partial \Omega_e}{\partial f/\partial E} \tag{28}$$

Using (25) as the implicit function, (26), (27) and (28) can be calculated from the partial derivatives of the function with respect to E, K, θ and Ω_e:

$$\partial f/\partial E=-\tfrac{1}{2}\theta^{1/2}E^{-1/2}\Omega_e+K\theta^{1/2}E^{-3/2}+K\theta E^{-2} \tag{29}$$

---

[8] According to the IFT, given that $f(x,y)$ is a function on a ball about (x_0, y_0) in R^2 and $f(x_0, y_0) = C$, if $\frac{\partial f}{\partial y}(x_0,y_0)\neq 0$ then there exists a function $y = y(x)$ such that: $y'(x_0)=-\frac{\partial f/\partial x}{\partial f/\partial y}$ (Simon and Blume, 1994).

$$\partial f/\partial E = \theta^{1/2}E^{-1/2}\left(K\theta^{1/2}E^{-3/2} + K/E - \Omega_e/2\right) \tag{29'}$$

$$\partial f/\partial K = -\left(1+\sqrt{\theta/E}\right)^2 \tag{30}$$

$$\partial f/\partial \theta = -\tfrac{1}{2}\theta^{-1/2}E^{1/2}\Omega_e - K\theta^{-1/2}E^{-1/2} - K/E \tag{31}$$

$$\partial f/\partial \Omega_e = \left(1-\sqrt{\theta E}\right) \tag{32}$$

Clearly, equations (30) and (31) are always negative, while (32) is always positive, given the parameter range described by (A1) (and for those ranges, a real root to (24) exists). However, the sign of *df/dE* depends on the RHS of (29'), and, more precisely, on the sign of the difference:

$$\frac{K}{E}\left(\sqrt{\theta/E}+1\right)-\frac{\Omega_e}{2} \tag{33}$$

The sign of equation (33) depends on how large E is:[9] if E is small, the expression is always positive (for any finite values of Ω_e and K); if E is large, the expression is

---

[9] It has to be said, however, that the sign of equation (33) could not only depend on how large the initial proportion of geniuses (E) is, but also on the initial values of the other three variables (Ω_e, K and θ). After all, the proportion of geniuses in equilibrium is a function of these variables. One of the aspects to consider is the relative size of the value of ideas (Ω_e) with respect to the investment cost (K). This is particularly relevant when E is medium-sized. Also, given equation (16), the proportion of geniuses (E) depends on the size of effectiveness of pirating (θ) and the guarding time (G): So, it can be argued that when the effectiveness of pirating is low and/or the guarding time is high, the initial proportion of geniuses (initial E) is low and, the equation (33) is positive.

always negative. Therefore, the sign of the derivative df/dE depends on the initial conditions of the economy, in terms of the existing proportion of geniuses:

$$\partial f/\partial E > 0 \qquad \text{For small values of } E \text{, and}$$

$$\partial f/\partial E < 0 \qquad \text{For large values of } E[10]$$

Proposition 1

The sign of the partial derivative of the implicit function with respect to the proportion of geniuses in the economy (df/dE) depends on the initial proportion of geniuses in the economy. If there are few geniuses initially, the derivative is always positive *for any finite values of Ω_e and K. On the other hand, if there are many geniuses in the economy, the derivative df/dE will be always* negative, *for any finite values of Ω_e and K. Moreover, there is a unique critical value for the proportion of geniuses such that df/dE is zero.*

Therefore, the impact of the investment cost, IPRs protection, and value of ideas, on the proportion of geniuses, dE/dK, $dE/d\theta$ and $dE/d\Omega_e$, will depend on the initial value of the proportion of geniuses (E) itself. In this sense, two different situations can arise: one for a positive RHS of (29) and the other when the RHS of (29) is negative.

If (29) is positive (small values of E), and given the signs of (30), (31) and (32):

---

[10] The ambiguous comparative statics arise because the parameter ranges of (A1), $\Omega_e > K > 0$ and $\sqrt{\theta E} < 1$ are not enough to determine whether the expression in parenthesis of (29') is positive or negative. Notice, however, that when $E \rightarrow 1/\theta$ (its largest value), $C_e - K > 0$ and $D > 0$, so the "large range" is consistent with (A1) and the splitting equilibrium condition ($C_e - K = D$). Also, if $E \rightarrow 0$ (its smallest value), $C_e - K > 0$ and $D > 0$, so the "small range" is also consistent with (A1) and the splitting equilibrium (see Appendix).

$$dE/dK > 0$$

$$dE/d\theta > 0$$

$$dE/d\Omega_e < 0$$

However, if (29) is negative (high E), and given the signs of (30), (31) and (32):

$$dE/dK < 0$$

$$dE/d\theta < 0$$

$$dE/d\Omega_e > 0$$

Lemma 1

The impact of Ω_e, K and θ on the proportion of geniuses depends on how many geniuses there are initially in the economy. If there are many geniuses in the economy, a decrease in the investment cost, a decrease in the effectiveness of pirating or an increase in the value of ideas created will raise *the proportion of geniuses. But, if there are few geniuses initially, a decrease in the investment cost, a decrease in the effectiveness of pirating or an increase in the value of ideas created will* reduce *the proportion of geniuses.*

There are two implications of Lemma 1. Firstly, Grossman's results of a negative impact of the effectiveness of pirating and a positive impact of the value of ideas created on the proportion of inventors ($dE/d\theta < 0$ and $dE/d\Omega_e > 0$) are true only in some cases: these derivatives will have the signs predicted by Grossman's model when the initial number of geniuses is large (high values of E). However, if there are few geniuses around initially, the impacts of the effectiveness of pirating and the value of ideas on the proportion of geniuses work in the opposite direction. This means that including an investment cost in Grossman's separating equilibrium creates an ambiguity in the results of the impact of policy variables on the proportion of inventors.

Secondly, an economy without geniuses starts always with a positive df/dE (equation 33 is positive) whatever levels of Ω_e and K (satisfying parameter ranges of A1). Then, when the proportion of geniuses in the economy is sufficiently large, the derivative df/dE changes signs and becomes negative (equation 33 is negative) for any finite values of Ω_e and K. Moreover, the sign of df/dE remains then negative for larger values of E, and never go back to a positive df/dE.

This means, that when an economy begins with a small proportion of geniuses (or with no geniuses at all), a *decrease* in the investment cost, a *decrease* in the effectiveness of pirating or an *increase* in the value of ideas created will *reduce* the proportion of geniuses. In this case, the proportion of pirates (R) is high (equation 15), the guarding time (G) is high (equation 16) and the proportion of the value of ideas in inventors' hands (p) is low (equation 17).

These conditions describe an economy where there are few ideas to be copied (few inventors and too much time spent in guarding) and the pirates' share of the value of ideas must be divided among many fellow pirates. Here, if the investment cost (K) increases, the effectiveness of pirating (θ) increases and the value of ideas created (Ω_e) decreases, there will be even less ideas to be copied, so the share of each pirate (one of many fellow pirates in the economy) is even smaller. Additionally, if we consider the proportion of the value of ideas in inventors' hands (p):

$$p = \frac{1}{1+\frac{\theta R}{G}} \tag{4}$$

Which can be rewritten as:

$$p = \frac{G}{G+\theta R} \tag{4'}$$

Or, from the results of the separating equilibrium:

$$p = \frac{1}{1+\sqrt{\theta/E}} \qquad (17)$$

If the proportion of geniuses (E) begins to increase (from a situation in which there are few geniuses around or no geniuses at all), the proportion of pirates (R) will be decreasing (by equation 15) and, therefore the proportion of the value of ideas in inventors' hands (p) will increase (by equation 4'). This is re-affirmed by equation (17): when E is increasing, the proportion of the value of ideas in inventors' hands (p) will increase as well. This situation means that when the (initially small) proportion of geniuses begins to increase, there will be more incentives to leave the pirates' sector and invest to become a genius (as the corresponding proportion of the value of ideas in that sector is increasing).

Then, when the proportion of geniuses is large enough (high values of E), there is a change of signs and the impacts of these variables work in the opposite direction: a *decrease* in the investment cost, a *decrease* in the effectiveness of pirating or an *increase* in the value of ideas created will *raise* the proportion of geniuses. In this case, the proportion of pirates (R) is low (equation 15), the guarding time (G) is low (equation 16) and the proportion of the value of ideas in inventors' hands (p) is high (equation 17).

Here, the high share of the value of ideas in inventors' hands is attractive for inventors, while a low guarding time implies more ideas to be copied and shared among few pirates (as R is low). Therefore, we have an economy with many incentives to become an inventor: there are few pirates, there is no need to allocate resources to guarding and the proportion of value of ideas in inventors' hands is high. Under these conditions, if the investment cost decreases and the IPRs protection increases, more individuals will be interested in investing to become geniuses.

Additionally, if we look at the proportion of the value of ideas in inventors' hands:

$$p = \frac{G}{G + \theta R} \tag{4'}$$

Or, from the results of the separating equilibrium:

$$p = \frac{1}{1 + \sqrt{\theta / E}} \tag{17}$$

We can see that once E is increasing (or, given equation 15, the proportion of pirates (R) is decreasing), both equations (4') and (17) reveal that the proportion of the value of ideas in inventors' hands are rising. This means that once the proportion of geniuses in the economy is large enough, as to change the sign of equation (33), it will keep increasing as there are more incentives to leave the pirates' sector and invest to become a genius (the proportion of the value of ideas as a genius is higher) and will never go back to a situation in which there are few geniuses around.

5. POLICY RECOMMENDATIONS

The impacts of the value of ideas, investment cost and effectiveness of pirating on the proportion of geniuses in the economy, described by Lemma 1, can be used to formulate policy recommendations within the model presented here.

Specifically, one could argue that the value of ideas created (Ω_e) is not a policy, but is given in any industry by market scientific conditions. Hence, let θ and K be policy variables. As θ is the effectiveness of pirating, a decrease in its value represents higher protection. In this sense, θ is considered an instrument for the authorities: a strong policy of IPRs protection will be translated into a lower value of θ. On the other hand, the investment cost (K) can be reduced by better training or education of the labor force, or simply covered (subsidized) by the authorities, so it can be considered as a second policy instrument.

For a given θ, when the economy starts with a high proportion of geniuses (E is large), a reduction of the investment cost (K) will be effective to raise the proportion of geniuses in the economy (for large E: $dE/dK < 0$, so as K decreases, E will increase). In this case, a reduction in the investment cost increases the incentives to invest in inventing; so more individuals (firms) will choose to become geniuses.

However, when the economy starts with a low proportion of geniuses (E is small), a reduction of K will be ineffective to raise the proportion of geniuses (for small E: $dE/dK > 0$, so as K decreases, E will decrease as well). In this case, an increase in the investment cost actually encourages inventions because, as there are few geniuses in the economy, there are too many fellow pirates sharing a small value of ideas (few ideas to copy).

Similarly, for a given investment cost (K), when the economy starts with a high proportion of geniuses (E is large), a higher IPRs protection (lower θ) will be effective to raise the proportion of geniuses in the economy (for large E: $dE/d\theta < 0$, so as θ decreases, E will increase). In this case, a stronger IPRs protection will reduce the already small guarding time (equation 16) and increase the already high proportion of ideas in inventors' hands (equation 17), so it will effectively raise the proportion of individuals (firms) that invest in inventing.

But, when the economy starts with a small proportion of geniuses (E is small), a strong IPRs protection (lower θ) will not be effective to raise the proportion of geniuses in the economy (for small E: $dE/d\theta > 0$, so as θ decreases, E will decrease as well), so the best policy on property rights will be a less protective one. In this second case, with few geniuses around, the proportion of pirates in the economy is high (equation 15) and the guarding time is high (equation 16), so there are few ideas to be copied. Even if the IPRs protection becomes weaker (the effectiveness of pirating is increased), there are more incentives to move out of the (less attractive) pirates' sector towards the inventors' sector.

Summarizing the policy recommendations described above, the results of the impacts of policy variables on the proportion of geniuses (stressed in Lemma 1) suggest that the effectiveness of policy instruments to raise the proportion of geniuses depends on the initial conditions of the economy, in terms of the existing proportion of geniuses itself.

When the economy starts with a high proportion of geniuses, a strong policy on IPRs protection and a policy on reducing the investment cost will effectively raise the proportion of geniuses in the economy. In this case, its already more attractive to become an inventor (lower guarding time and higher proportion of the ideas in inventors' hands) and a reduction in the investment cost or a stronger protection against pirating creates more incentives to become an inventor.

However, when the economy starts with few geniuses, the best policy of IPRs protection is a less protective one. In this second case, there are too many pirates around that have to share a proportion of the value of very few ideas created. A weaker IPRs protection will not create incentives to remain in the pirates' sector: it helps to become a pirate, but an increase in the proportion of pirates (sharing little value of ideas among many fellow pirates) create incentives to move out of this sector and invest to become an inventor.

6. WELFARE ANALYSIS

So far, our model predicts that for some values of ideas created, costs of investment and IPRs protection, the proportion of geniuses in the economy will increase. In particular, it has been suggested that for industries with a large initial proportion of geniuses, the proportion of geniuses will increase with the value of ideas created and with a stronger IPRs protection, while it will decrease for higher values of the investment cost. Meanwhile, for industries with a low initial proportion of geniuses, the impacts of these variables will work in the opposite direction.

However, nothing has been said about the advantages that an increase in the proportion of geniuses represents for the society. Is it always in the interest of the society to have a high proportion of geniuses or could it be that in some cases there is more welfare for a small proportion of geniuses? These questions are addressed in the present section.

In the framework presented in the second section, there are two types of individuals (firms) in the society: geniuses-inventors and ordinary people-pirates. In order to construct a welfare function for this type of society, the benefits of each group will be added and the cost paid (in this case, the investment cost paid by geniuses) must be deducted. In this sense, a simple welfare function is proposed:

$$W = eC_e + (1-e)D - eK \tag{34}$$

The first issue to be mentioned about this function is that the proportion of geniuses in the economy can have two types of impacts on welfare: a direct (by the change in e itself) and an indirect one (through the change in C_e or D). According to equation (20), we must always keep the equality $C_e - K = D$ in order to remain in the splitting equilibrium. This situation implies that the direct impact of e on W (on equation (34)) is not relevant, as the balance of wealth for inventors-pirates always holds. In this sense, only the indirect effect should be considered.

As our interest is to derive the welfare function for the splitting equilibrium only, we can substitute the equality $C_e - K = D$ into equation (34).[11] The wealth function yields:

$$W = D \tag{35}$$

Equation (35) suggests that for a split-then-separate type of economy, society's welfare can be calculated using the wealth of only one of the two parties involved; in this case, pirates' wealth (D). The logic behind this result is that for a split-then-separate type of economy, the wealth of both parties must be

[11] Equation (34) can be rewritten as $W = e(C_e - K) + (1-e)D$, where $C_e - K = D$ for the splitting equilibrium.

always balanced ($C_e - K = D$) to remain in equilibrium. In this sense, it is enough to know the wealth of one of the parties, in order to calculate society's wealth.

Therefore, to obtain the sign of the increase in the proportion of geniuses on society's welfare (*i.e.* the indirect effect, the impact of E on D and the subsequent impact of D on W) we can use equation (35) and derive D with respect to E.

We know that for the separating equilibrium:

$$D = \frac{\sqrt{\theta E}\,\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2} \tag{19'}$$

Then, the derivative of D with respect to E is:

$$\frac{dD}{dE} = \frac{\tfrac{1}{2}\theta^{1/2}E^{-1/2}\Omega_e\left(1+\theta^{1/2}E^{-1/2}\right)^2 - 2\left(1+\theta^{1/2}E^{-1/2}\right)\left(-\tfrac{1}{2}\theta^{1/2}E^{-3/2}\right)\theta^{1/2}E^{1/2}\Omega_e}{\left(1+\sqrt{\theta/E}\right)^4} \tag{36}$$

Which can be reduced to:

$$\frac{dD}{dE} = \frac{\Omega_e\left(\sqrt{\theta/E}\right)}{2\left(1+\sqrt{\theta/E}\right)^2} + \frac{\Omega_e\left(\theta/E\right)}{\left(1+\sqrt{\theta/E}\right)^3} \tag{36'}$$

And which is clearly positive.

Therefore, it can be seen, from the sign of (36'), that there is a positive (indirect) impact of E on W: as the proportion of geniuses increases, the wealth of pirates (equivalent to inventors' wealth in equilibrium) will increase, and so will the welfare.[12]

The intuition for this positive (indirect) impact can be obtained from equation (19):

$$D = \frac{(1-p)}{R} \frac{\Omega_e}{1+G} \tag{19}$$

If there is an increase in E, there is a positive impact on D. There are three effects (two positive and one negative) of the increase of E on pirates' wealth (D). First, the pirates' share of the value of ideas, $(1-p)$, will decrease as E increases (equation 17), but this negative effect is cancelled out by a (larger) positive effect of E on the proportion of pirates (R): as the number of geniuses in the society increases, the number of pirates will decrease in a one-to-one proportion (equation 15). Additionally, there is a third (and positive) impact of E on D: the guarding time (G) decreases as E increases (equation 16): as there are fewer pirates, less effort will be done on guarding.

The positive impact of E on W and equality of $C_e - K = D$ imply that an increase in the proportion of geniuses will be beneficial for both groups, geniuses and pirates, and the society will be better off.[13]

Substituting (18') for inventors' wealth and (19') for pirates' wealth in (34):

---

[12] This result implies that when E increases, the (splitting equilibrium) equality $C_e - K = D$ still holds, but for higher values of C_e and D. In other words, if the equality always holds, it should not matter to welfare how many pirates or inventors there are in the economy (the payoff to both is the same), unless such payoff is increased for both groups.

[13] However, it has to be said that, while this result seems to be quite straightforward, it is a partial one, as only the splitting equilibrium is analyzed here.

$$W = e\left(\frac{\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2}\right) + (1-e)\left(\frac{\sqrt{\theta E}\,\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2}\right) - eK \tag{37}$$

Rearranging terms in (37), the welfare function can be reduced to a more manageable expression, for welfare in terms of the proportion of geniuses (e):[14]

$$W = \frac{e\Omega_e}{1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}} - eK \tag{38}$$

And, given that the optimal proportion of geniuses in (38) is a function of several variables (Ω_e, θ and K), a more formal expression for W would be:

$$W = \frac{e(\Omega_e,\theta,K)\Omega_e}{1+e(\Omega_e,\theta,K)^{-1/2}[1-e(\Omega_e,\theta,K)]^{1/2}\theta^{1/2}} - e(\Omega_e,\theta,K)K \tag{38'}$$

Once this reduced welfare expression has been found, comparative statics can be obtained to determine the impact of an increase in the value of ideas created (Ω_e), the effectiveness of pirating (θ) and the investment cost (K) on society's welfare. To do so, derivatives for $dW/d\Omega_e$, $dW/d\theta$, dW/dK are obtained from (38'). These derivatives can be expressed as a sum of direct and indirect effects on welfare. The direct effect represents the impact of Ω_e, θ or K on W, while the indirect effect measures the impact of Ω_e, θ or K on the proportion of geniuses (e) and the subsequent effect of this proportion on welfare (given that the impact of e on W is via a change in C_e - K or D).

These comparative statics are relevant to know the impact of policy variables on welfare. As discussed before, there are several possible policy variables

---

[14] The reduction of equation (37) and derivation of (38) are included in the Appendix.

in our model. The first one is the IPRs protection (θ), stressed by Grossman's article. New laws on IPRs protection could be proposed based on the impact of the effectiveness of pirating (θ) on welfare. Additionally, an alternative (or complementary) policy could be analyzed: a policy of subsidies on the cost of investment, K (in the form of education or training). These policies can be discussed after the impact of these variables on welfare is known.

The impact of the value of ideas (Ω_e) on welfare can be obtained from the derivative:

$$\frac{dW}{d\Omega_e}=\frac{\partial W}{\partial \Omega_e}+\left[\frac{\partial W}{\partial e}\right]\frac{de}{d\Omega_e} \tag{39}$$

$$\frac{dW}{d\Omega_e}=\frac{e}{\left(1+\sqrt{\theta/E}\right)}+\left[\frac{\Omega_e}{1+\sqrt{\theta/E}}-K+\frac{\frac{\Omega_e}{2}\left[\sqrt{\theta E}+\sqrt{\theta/E}\right]}{\left(1+\sqrt{\theta/E}\right)^2}\right]\frac{de}{d\Omega_e} \tag{39'}$$

The first term of (39') reflects the direct effect of the value of ideas (Ω_e) on welfare, while the second term considers the indirect effect: it combines the impact of the value of ideas (Ω_e) on the proportion of geniuses and the impact of the function $e(\Omega_e$, θ and $K)$ on W. The sign of the derivative (and, therefore, the total impact on W) depends on the signs of these two effects, the direct and indirect ones.

The direct effect is positive, as the proportion of geniuses is always positive. If you have more valuable ideas in the industry, all else equal, welfare would rise. However, as the indirect effect depends on the impact of the value of ideas (Ω_e) on the proportion of geniuses ($de/d\Omega_e$), its sign is ambiguous. The expression in parenthesis (dW/de) is always positive[15] for all $\Omega_e > K > 0$, and

[15] The difference between the first two terms in the parenthesis is always positive given equation (21). If the investment cost (K) was larger than the first term, nobody would invest in becoming a genius.

$\sqrt{\theta E} < 1$. As mentioned before, all else equal, the more geniuses in the economy, the higher the pirates' wealth and, subsequently, through the indirect effect, the higher the society's welfare. Meanwhile, the sign of the derivative of the proportion of geniuses with respect to the value of ideas ($de/d\Omega_e$) depends on the initial proportion of geniuses (as stressed before).

For industries characterized by a large initial proportion of geniuses, the proportion of geniuses increases as the value of ideas increases, *i.e.* $dE/d\Omega_e > 0$. In this case, both the first and second terms of (39') are positive, so both the direct and the indirect effects of the value of ideas on welfare are positive. In other words, for industries with high E, an increase in the value of ideas will raise society's welfare.

On the other hand, for industries characterized by a small initial proportion of geniuses, the proportion of geniuses decreases as the value of ideas increases, *i.e.* $dE/d\Omega_e < 0$. In this case, the second term of (39') is negative, while the first term is positive: the direct and indirect effects on W work in opposite directions. In other words, for industries with small E, the effect of an increase in the value of ideas on society's welfare depends on which effect dominates.

The impact of the IPRs protection (θ) on welfare can be obtained from the derivative:

$$d\frac{dW}{d\theta}=\frac{\partial W}{\partial \theta}+\left[\frac{\partial W}{\partial e}\right]\frac{de}{d\theta} \tag{40}$$

$$\frac{dW}{d\theta}=-\frac{\frac{\Omega_e}{2}\left(e^{1/2}(1-e)^{1/2}\theta^{-1/2}\right)}{\left(1+\sqrt{\theta/E}\right)^2}+\left[\frac{\Omega_e}{1+\sqrt{\theta/E}}-K+\frac{\frac{\Omega_e}{2}\left[\sqrt{\theta E}+\sqrt{\theta/E}\right]}{\left(1+\sqrt{\theta/E}\right)^2}\right]\frac{de}{d\theta} \tag{40'}$$

The first term of (40') reflects the direct effect of the IPRs protection (θ) on welfare, while the second term considers the indirect effect: it combines the impact of the IPRs protection (θ) on the proportion of geniuses and the impact of the function e (Ω_e, θ and K) on W. The sign of the derivative (and, therefore, the total impact on W) depends, once again, on the signs of these two effects, the direct and indirect ones.

The direct effect is negative for all $\Omega_e > K > 0$ and $\sqrt{\theta E} < 1$. If all else equal, the tougher the IPRs protection (the smaller the effectiveness of pirating, θ), the lower the guarding time, G (equation 16), the higher the inventors' and pirates' wealth (equations 18 and 19, respectively), and the higher the society's welfare. However, as the indirect effect depends on the impact of θ on the proportion of geniuses ($dE/d\theta$), its sign is ambiguous.

The expression in parenthesis is again always positive[16] for all $\Omega_e > K > 0$ and $\sqrt{\theta E} < 1$. All else equal, the more geniuses in the economy, the higher the pirates' wealth and, subsequently, through the indirect effect, the higher the society's welfare. Meanwhile, the sign of the derivative of the proportion of geniuses with respect to the IPRs protection ($dE/d\theta$) depends on the initial conditions of the industry (as stressed before).

For industries characterized by a large proportion of geniuses, the proportion of geniuses increases as the IPRs protection is tougher (smaller θ), *i.e.* $dE/d\theta < 0$. In this case, when θ increases (less protection), both, the first and the second terms of (40') are negative, so the direct and the indirect effects of less IPRs protection on welfare are negative. In other words, for industries with many geniuses, an increase in the IPRs protection will raise society's welfare: a tougher IPRs protection will reduce the guarding time (equation 16) and increase the proportion of the value of ideas in inventors' hands (equation 17), so it will raise the proportion of people (firms) who invest in inventing, which, in turn, improves pirates' wealth too, by giving them more to copy (less

[16] The expression in parenthesis is, in fact, the same as the one in equation (39'), so the same intuition for this positive sign applies here.

guarding time) and reducing the proportion of fellow pirates (equation 15) with whom the value of ideas created is shared.

On the other hand, for industries characterized by a small proportion of geniuses, the proportion of geniuses decreases as the protection is tougher, *i.e.* $dE/d\theta > 0$. In this case, the second term of (40') is positive, while the first term is negative: once again, the direct and indirect effects on W work in opposite directions. In other words, for industries with few geniuses, the effect of an increase in IPRs protection on society's welfare depends on which effect dominates. Therefore, it could very well be the case that stronger IPRs protection has the perverse effect of lowering the inventiveness of the society. As the initial proportion of geniuses is small, the guarding time is high (equation 16) and the proportion of the value of ideas in inventors' hands is low (equation 17), so there are fewer incentives to invest in becoming an (genius) inventor.

The impact of the investment cost (K) on welfare can be obtained from the derivative:

$$\frac{dW}{dK} = \frac{\partial W}{\partial K} + \left[\frac{\partial W}{\partial e}\right]\frac{de}{dK} \tag{41}$$

$$\frac{dW}{dK} = -e + \left[\frac{\Omega_e}{1+\sqrt{\theta/E}} - K + \frac{\frac{\Omega_e}{2}\left[\sqrt{\theta E} + \sqrt{\theta/E}\right]}{\left(1+\sqrt{\theta/E}\right)^2}\right]\frac{de}{dK} \tag{41'}$$

The first term of (41') reflects the direct effect of the investment cost (K) on welfare, while the second term considers the indirect effect: it combines the impact of the investment cost (K) on the proportion of geniuses and the impact of the function $e(\Omega_e$, θ and $K)$ on W. The sign of the derivative depends on the signs of direct/indirect effects.

The direct effect is negative, as the proportion of geniuses (e) is always positive. All else equal, raising the investment cost cannot raise society's welfare.

However, as the indirect effect depends on the impact of K on the proportion of geniuses (dE/dK), its sign is ambiguous. The expression in parenthesis is again always positive for all $\Omega_e > K > 0$ and $\sqrt{\theta E} < 1$. All else equal, the more geniuses in the economy, the higher the pirates' wealth and, subsequently, through the indirect effect, the higher the society's welfare. Meanwhile, the sign of the derivative of the proportion of geniuses with respect to the investment cost (dE/dK) depends on the initial conditions of the industry (as stressed before).

For industries characterized by a large proportion of geniuses, the proportion of geniuses decreases as the investment cost increases, *i.e.* $dE/dK < 0$. In this case, both the first and the second terms of (41') are negative, so both the direct and indirect effects of the investment cost on welfare are negative. In other words, for industries with many geniuses, an increase in the investment cost will decrease society's welfare, as it reduces the incentives to invest in inventing, which has a negative effect on pirates (less ideas to be copied and more fellow pirates (equation 15) with whom you have to share the proportion of the value of ideas) as well.

On the other hand, for industries characterized by a small proportion of geniuses, the proportion of geniuses increases as the investment cost increases, *i.e.* $dE/dK > 0$. In this case, the second term of (41') is positive, while the first term is negative: the direct and indirect effects on W work in opposite directions. In other words, for industries with few geniuses, the impact of an increase in the investment cost on society's welfare depends on which effect dominates. Here, an increase in the investment cost actually encourages inventions because there are too many fellow pirates sharing a small value of ideas (few ideas to copy), and the increase in the investment cost leaks out, as the guarding time decreases (equation 16) and the proportion of ideas in inventors' hands increases (equation 17) as soon as people (firms) move from pirates to inventors (*i.e.* E increases).

The comparative statics described by derivatives (39'), (40') and (41') can be summarized in Table 1.

The results of the comparative statics suggest that, as in the previous section, the impact of the policy variables on welfare depend on the initial proportion of

TABLE 1

INITIAL VALUE / VARIABLE	EFFECT ON WELFARE		
	Direct	Indirect	Total
Large E			
Ω_e	+	+	+
θ	-	-	-
K	-	-	-
Small E			
Ω_e	+	-	?
θ	-	+	?
K	-	+	?

geniuses. For industries characterized by a high initial proportion of geniuses, the proportion of geniuses and society's welfare will increase when the value of ideas increases, when the investment cost decreases and when the effectiveness of pirating decreases. In these industries, a tough IPRs protection (low θ), as well as a policy on the investment cost (K), will not only have a positive impact on the proportion of geniuses in the economy, but it also will raise society's welfare.

However, for industries with a low initial proportion of geniuses, the results are less straightforward. The impact of the independent variables (Ω_e, θ and K) on the proportion of geniuses work on the opposite direction as in the other type of industry (high E): the proportion of geniuses increases with a lower value of ideas, a higher value of the investment cost and a less tough IPRs protection policy. Additionally, the impact of these variables on society's welfare is ambiguous, as it depends on which effect dominates: the direct (the impact of Ω_c, θ and K on W) or the indirect one (the impact of Ω_c, θ and K on the proportion of geniuses and the subsequent impact of $e(\cdot)$ on W).

7. CONCLUSIONS

The theoretical model presented in this paper provides insight for the particular structure of a dual economy. An inventor-pirate framework (in particular, the model presented by Grossman, 2005) was used to observe a previous stage, in which an individual (firm) has to decide whether to invest and become an inventor (genius) or to remain as a pirate.

The model presented here focuses on only one of the four equilibria proposed by Grossman's article: the separating equilibrium, in which all geniuses are inventors and all ordinary people are pirates. This implies that in equilibrium those that have chosen to invest and become geniuses will be inventors, while those that have chosen to remain ordinary will become pirates and will, thereafter, copy others' ideas. This equilibrium is combined with a splitting type of economy in the first stage, where some individuals (firms) choose to become geniuses while some prefer to remain as ordinary ones.

There is a clear division in the society for this type of equilibrium: those individuals that invest in education/training are able to develop their own inventions, while the rest of the society is only able to imitate those inventions. Many economies have this type of structure: there is a clear division between those sectors closely related to advanced technology (that are continuously learning how to develop new ideas) and those that are not familiar with this type of technology (and remain as imitators of other's inventions).

The main result of the model presented here is that, even when a higher proportion of geniuses represent a higher welfare for the society, the policy implemented by the authorities to create incentives for such an increase depends on the initial conditions of the economy (in terms of the proportion of geniuses itself).

The results obtained here suggest that there are two contrasting situations in this type of economy. For industries with many geniuses, the proportion of geniuses (and welfare) will increase with the value of ideas created and decrease with the effectiveness of pirating and cost of investment, and a policy of strong IPRs protection will be effective in increasing the proportion of geniuses and welfare. With many geniuses there are many ideas (with high value) and the geniuses' share is high; if a decrease in the effectiveness of pirating or the investment cost is added, it is even more attractive to become a genius.

Meanwhile, for industries characterized by a low number of geniuses, the proportion of geniuses will increase with the effectiveness of pirating and the cost of investment, and decrease with the value of ideas created. In this case, there are few ideas to be copied and pirates' share is divided among many fellow pirates. There are more incentives to move out of the pirate sector, even if the cost of investment increases or IPRs protection decreases. Hence, the best IPRs protection policy is a less strong one. Additionally, in these industries, the impact of policy variables on welfare is ambiguous; as it depends on which effect (direct or indirect one) dominates.

The ambiguous comparative statics could be explained by the fact that different industries tend to have different innovative conditions, even in developed economies. In particular, one can think of contrasting industries in terms of value of ideas and investment cost, as pharmaceuticals (high costs of investment and low value of ideas created) and software (low costs of investment and high value of ideas) ones.

Pharmaceutical industry is characterized by high costs of innovation (on a trial-and-error basis), which have increased in recent years.[17] On the other hand, the possibility of copies for new products and the fact that the introduction of those new products (due to new technologies at the global level) is slowing lately makes it uncertain and difficult to have high rewards to innovation (Matraves, 1999; Gonsen and Jasso, 2000).

[17] A recent empirical study by DiMasi, *et al.* (2002) has shown that R&D costs of new drugs have increased at an annual rate of 7.4% above general price inflation.

Meanwhile, the software industry is characterized by low marginal costs and rapid technological change: after the cost of design and develop of a new product, the marginal cost of producing another copy is minimal. The value of the developed software is high, as there is a captive market of consumers, as all software products are differentiated and the consumers prefer to acquire products that are compatible with her existing software (Klemperer, 1987; Schmalensee, 2000). This type of analysis could be conducted in more detail in future studies.

Finally, although it could be tempting to apply the present model to the other equilibria proposed by Grossman's article (the pooling ones: not all geniuses are inventors and not every ordinary person is a pirate), the (splitting-then-separating) equilibrium analyzed here seems to be the one that best fits to many developing economies as Mexico.

REFERENCES

Arjona, L. and K. Unger (1996), "Competitividad internacional y desarrollo tecnológico: la industria manufacturera mexicana frente a la apertura comercial", *Economía Mexicana*, vol. V (2), 187-220.

Becker, G. (1968), "Crime and punishment: an economic approach", *Journal of Political Economy*, 169-217.

Becker, G. and G. Stigler (1974), "Law Enforcement, Malfeasance and Compensation of Enforcers", *Journal of Legal Studies* 3, 1-18.

Brown, F. and L. Dominguez (1999), *Productividad: desafío de la industria mexicana*, JUS, UNAM, México.

Casar, J.I. (1993), "La competitividad de la industria manufacturera mexicana 1980-1990", *El Trimestre Económico*, vol. LX (1), num. 237, 113-183.

Cimoli, M. (ed.) (2000), *Developing Innovation Systems: Mexico in a Global Context*, The Continuum International Publishing Group, London, U.K.

Cozzi, G. (2001), "Inventing or Spying? Implications for Growth", *Journal of Economic Growth*, 6 (1), March, 55-77.

Di Masi, J.A., R.W. Hansen and H.G. Grabowski (2002), "The price of innovation: new estimates of drug development costs", *Journal of Health Economics*, Vol. 22, No.2.

Dussel, E. (1994), "Cambio estructural y potencialidades de crecimiento del sector manufacturero en México, 1982-1991" in J. López (ed.) *México: la nueva macroeconomía*, Nuevo Horizonte, México.

Dosi, G., C. Freeman, R. Nelson and L. Soete (1988), *Technical Change and Economic Theory*, Pinter, London, U.K.

Friedman, D. (1984), "Efficient Institutions of the Private Enforcement of Law", *Journal of Legal Studies* 13, 379-97.

Fujii, D. (2004), "La tecnología y el éxito industrial en México: una propuesta de división sectorial", *Revista de Economía Mundial* 10-11, 105-126.

Gonsen, R. and J. Jasso (2000), "The Pharmaceutical Industry" in Cimoli (ed.) *Developing Innovation Systems*, Continuum, London, U.K.

Grossman, H.I. (1998), "Producers and Predators", *Pacific Economic Review*, 3(3), October, 169-187.

Grossman, H.I. (2005), "Inventors and pirates: creative activity and intellectual property rights", *European Journal of Political Economy*, 21, 269-285.

Grossman, H.I. and M. Kim (1995), "Swords or Plowshares? A Theory of the Security of Claims to Property", *The Journal of Political Economy*, 103 (6), December, 1275-1288.

Grossman, H.I. and M. Kim (1996), "Predation and Production", in M.R.Garfinkel and S. Skaperdas (eds), *The Political Economy and Appropriation*, Cambridge University Press.

Grossman, H.I. and M. Kim (2002), "Predation, Efficiency and Inequality", *Journal of Institutional and Theoretical Economics*, 158 (3), September, 393-407.

Grossman, H.I. and M. Kim (2003), "Education policy: egalitarian or elitist?", *Economics and Politics*, 15 (3), November, 225-245.

Klemperer, P. (1987), "Markets with consumer switching costs", *Quarterly Journal of Economics*, vol.102, no.2, May, 375-394.

Landes, W.M. and R.A. Posner (1975), "The Private Enforcement of Law", *Journal of Legal Studies* 1.

Matraves, C. (1999), "Market structure, RandD and advertising in the pharmaceutical industry", *The Journal of Industrial Economics*, vol. XLVII (2), June, 169-194.

Neher, P.A. (1978), "The Pure Theory of the Muggery", *American Economic Review*, 437-445.

Schmalensee, R. (2000), "Antitrust Issues in Schumpeterian Industries", *American Economic Review*, vol.90, no.2, May, 192-196.

Simon, C.P. and S. Blume (1994), *Mathematics for Economists*, Norton and Co, New York.

Usher, D. (1987), "Theft as a paradigm for departures from efficiency", *Oxford Economic Papers*, 39, June 235-252.

APPENDIX

A cubic equation for E^* (equation 23):

$$\frac{\Omega_e}{(1+\sqrt{\theta/E})^2} - K = \frac{\sqrt{\theta E}\,\Omega_e}{(1+\sqrt{\theta/E})^2} \tag{22}$$

$$\frac{(1-\sqrt{\theta E})}{(1+\sqrt{\theta/E})^2} = \frac{K}{\Omega_e} \tag{23}$$

$$\Omega_e(1-\sqrt{\theta E}) = K(1+\sqrt{\theta/E})^2$$

$$\Omega_e(1-\sqrt{\theta E}) = K(1+2\sqrt{\theta/E}+\theta/E)$$

$$\Omega_e - \Omega_e\sqrt{\theta}E^{1/2} = K + 2K\sqrt{\theta}E^{-1/2} + K\theta E^{-1}$$

$$(\Omega_e - K) = \Omega_e\sqrt{\theta}E^{1/2} + 2K\sqrt{\theta}E^{-1/2} + K\theta E^{-1}$$

Then, we can assume that $\Omega_e > K > 0$, for a non-negative solution of E^*.

$$(\Omega_e - K) = \sqrt{\theta}\left[\Omega_e E^{1/2} + 2KE^{-1/2}\right] + K\theta E^{-1}$$

$$(\Omega_e - K) = \sqrt{\theta}\left[\frac{\Omega_e E + 2K}{E^{1/2}}\right] + K\theta E^{-1}$$

$$\frac{(\Omega_e - K)}{\theta K} = \theta^{-1/2}K^{-1}\left[\frac{\Omega_e E + 2K}{E^{1/2}}\right] + E^{-1}$$

$$\frac{E(\Omega_e - K)}{\theta K} = E^{1/2}\left[\frac{\Omega_e E}{\theta^{1/2}K} + \frac{2}{\theta^{1/2}}\right] + 1$$

We can express this formula as a cubic equation for $E^{1/2}$:

$$\left(\frac{\Omega_e}{\theta^{1/2}K}\right)E^{3/2}-\left(\frac{\Omega_e-K}{\theta K}\right)E+\left(\frac{2}{\theta^{1/2}}\right)E^{1/2}+1=0 \qquad (24)$$

Or, to simplify the expression, we can set: $X=E^{1/2}$

Then,

$$\left(\frac{\Omega_e}{\theta^{1/2}K}\right)X^3-\left(\frac{\Omega_e-K}{\theta K}\right)X^2+\left(\frac{2}{\theta^{1/2}}\right)X+1=0 \qquad (24')$$

An interior solution for E^*:

E^* can be obtained solving equation (24') for X^*. Recall that $X=E^{1/2}$.

There are 3 solutions to the cubic equation (24'), two imaginary and a real one. As we are interested in the proportion of geniuses in equilibrium, we will focus only on the last one. The (real) solution for X^* is:

$$X^*=-\frac{-\Omega_e+K}{3\Omega_e\theta^{1/2}}$$

$$-\frac{\left(2^{1/3}\left(6\Omega_e\theta K-(-\Omega_e+K)^2\right)\right)}{3\Omega_e\theta^{1/2}\left(-27\Omega_e^2\theta^2K+18\Omega_e\theta K(-\Omega_e+K)-2(-\Omega_e+K)^3+3\sqrt{3}\sqrt{\bullet}\right)^{1/3}}$$

$$+\frac{1}{3\cdot 2^{1/3}\Omega_e\theta^{1/2}}\left[\left(-27\Omega_e^2\theta^2K+18\Omega_e\theta K(-\Omega_e+K)-2(-\Omega_e+K)^3+3\sqrt{3}\sqrt{\bullet}\right)^{1/3}\right]$$

Where:

$$\sqrt{\bullet}=\sqrt{\left(-4\Omega_e^3\theta^2K+8\Omega_e^4\theta^2K^2+36\Omega_e^3\theta^3K^2+27\Omega_e^4\theta^4K^2-4\Omega_e^3\theta^2K^3-4\Omega_e^3\theta^3K^3\right)}$$

By assumption, we know that:

$$\Omega_e > K$$

Which, in turn, implies that:

$$(-\Omega_e + K) < 0$$

Additionally, as it is a real (not an imaginary) solution:

$$(-27\Omega_e^2\theta^2 K + 18\Omega_e\theta K(-\Omega_e + K) - 2(-\Omega_e + K)^3 + 3\sqrt{3}\sqrt{\bullet})^{1/3} > 0$$

And

$$\sqrt{\bullet} > 0$$

Also, we know that the equilibrium proportion of geniuses (E^*) is positive as:

$$X^* = (E^*)^{1/2}$$

Which means that:

$$E^* = X^2$$

So, even if the solution for X is negative (*i.e.* $X^* < 0$), E^* will be positive.
Therefore,

$$E^* > 0 \ \forall \Omega_e > K > 0 \quad \text{and} \quad \sqrt{\theta E} < 1 \qquad \text{(A1)}$$

Parameter ranges for "Large" E:

The sign of the derivative df/dE depends on the sign of the difference:

$$\frac{K}{E}\left(\sqrt{\theta/E}+1\right)-\frac{\Omega_e}{2} \tag{33}$$

Then,

$\partial f/\partial E < 0$ For large values of E

The maximum value of E under (A1) is:

$$E \to \frac{1}{\theta}$$

"Large"

Substituting the maximum value of E "Large" into (33):

$$2K\theta\left(\theta+1\right) < \Omega_e \tag{33'}$$

$$\theta\left(\theta+1\right) < \frac{\Omega_e}{2K} \tag{33''}$$

For the maximum range, E needs to satisfy:

(a) Equation (33")

(b) $\frac{\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2}-K>0$ $(C_e - K > 0)$

(c) $$\frac{\sqrt{\theta E}\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2} > 0 \qquad (D > 0)$$

For a large range of *E*, substituting "Large" into (b) and (c):

(a) Is satisfied for small values of θ: given A1, if *E* is large, θ has to be small

(b) $\frac{\Omega_e}{(1+\theta)^2} > K$ Is satisfied: given A1, if *E* is large, θ has to be small, and $\Omega_e > K$

(c) $\frac{\Omega_e}{(1+\theta)^2} > 0$ Is always satisfied for any positive value of Ω_e and θ

Parameter ranges for "Small" *E*:

The sign of the derivative *df/dE* depends on the condition:

$$\frac{K}{E}\left(\sqrt{\theta/E}+1\right)-\frac{\Omega_e}{2} \S 0 \qquad (33)$$

Then,

$$\partial f/\partial E > 0 \qquad \text{For small values of } E$$

The minimum value of *E* under (A1) is:

$$E \rightarrow 0$$

"Small"

For the minimum range, E needs to satisfy:

(a) $$\frac{K}{E}\left(\sqrt{\theta/E}+1\right)>\frac{\Omega_e}{2}$$

(b) $$\frac{\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2}-K>0 \qquad (C_e - K > 0)$$

(c) $$\frac{\sqrt{\theta E}\,\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2}>0 \qquad (D>0)$$

For a large range of E, substituting "Small" into (a), (b) and (c):

(a) Is always satisfied, as: $\infty > \frac{\Omega_e}{2}$

(b) Is satisfied given (A1), if E is small, θ has to be large and $\Omega_e > K$

(c) Is satisfied for all $\Omega_e > 0$

In sum:

When $E \rightarrow 1/\theta$ (its largest value), $C_e - K > 0$ and $D > 0$, so the "large range" is consistent with (A1) and the splitting equilibrium condition ($C_e - K = D$).

When $E \rightarrow 0$ (its smallest value), $C_e - K > 0$ and $D > 0$, so the "small range" is also consistent with (A1) and the splitting equilibrium.

An expression for W (equation 38):

$$W = eC_e + (1-e)D - eK \qquad (34)$$

Substituting C_e from (18') and D from (19') in (34):

$$W = e\left(\frac{\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2}\right) + (1-e)\left(\frac{\sqrt{\theta E}\,\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2}\right) - eK \quad (37)$$

$$W = \frac{e\Omega_e\left(1-\sqrt{\theta E}\right)+\Omega_e\sqrt{\theta E}}{\left(1+\sqrt{\theta/E}\right)^2} - eK$$

$$W = \frac{e\Omega_e\left(1-\theta^{1/2}E^{1/2}\right)+\Omega_e\theta^{1/2}E^{1/2}}{\left(1+\sqrt{\theta/E}\right)^2} - eK$$

$$W = \frac{e\Omega_e\left(1-\theta^{1/2}e^{1/2}/(1-e)^{1/2}\right)+\Omega_e\left(\theta^{1/2}e^{1/2}/(1-e)^{1/2}\right)}{\left(1+\sqrt{\theta/E}\right)^2} - eK$$

$$W = \frac{e\Omega_e + e^{1/2}(1-e)^{1/2}\theta^{1/2}\Omega_e}{\left(1+\sqrt{\theta/E}\right)^2} - eK$$

$$W = \frac{e\Omega_e\left(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)}{\left(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)^2} - eK$$

$$W = \frac{e\Omega_e}{1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}} - eK \quad (38)$$

And, given that optimal proportion of geniuses depends on Ω_e, θ and K:

$$W = \frac{e(\Omega_e,\theta,K)\Omega_e}{1+e(\Omega_e,\theta,K)^{-1/2}[1-e(\Omega_e,\theta,K)]^{1/2}\theta^{1/2}} - e(\Omega_e,\theta,K)K \qquad (38')$$

Equation (39'):

$$W = \frac{e(\Omega_e,\theta,K)\Omega_e}{1+e(\Omega_e,\theta,K)^{-1/2}[1-e(\Omega_e,\theta,K)]^{1/2}\theta^{1/2}} - e(\Omega_e,\theta,K)K \qquad (38')$$

$$\frac{dW}{d\Omega_e} = \frac{\left(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)\left[e+\Omega_e\frac{de}{d\Omega_e}\right] - (e\Omega_e)\left[-\tfrac{1}{2}e^{-1/2}(1-e)^{-1/2}\theta^{1/2} - \tfrac{1}{2}e^{-3/2}(1-e)^{1/2}\theta^{1/2}\right]\frac{de}{d\Omega_e}}{\left(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)^2}$$

$$-K\frac{de}{d\Omega_e}$$

$$\frac{dW}{d\Omega_e} = \frac{e\left(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)}{\left(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)^2}$$

$$+\left[\frac{\Omega_e}{\left(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)} - K + \frac{\frac{\Omega_e}{2}\left(e^{1/2}(1-e)^{-1/2}\theta^{1/2} + e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)}{\left(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)^2}\right]\frac{de}{d\Omega_e}$$

$$\frac{dW}{d\Omega_e} = \frac{e}{\left(1+\sqrt{\theta/E}\right)} + \left[\frac{\Omega_e}{1+\sqrt{\theta/E}} - K + \frac{\frac{\Omega_e}{2}\left[\sqrt{\theta E}+\sqrt{\theta/E}\right]}{\left(1+\sqrt{\theta/E}\right)^2}\right]\frac{de}{d\Omega_e} \qquad (39)$$

Equation (40'):

$$W = \frac{e(\Omega_e,\theta,K)\Omega_e}{1+e(\Omega_e,\theta,K)^{-1/2}[1-e(\Omega_e,\theta,K)]^{1/2}\theta^{1/2}} - e(\Omega_e,\theta,K)K \qquad (38')$$

$$\frac{dW}{d\theta} = \frac{(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2})\Omega^e \frac{de}{d\theta}}{(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2})^2}$$

$$\frac{-(e\Omega_e)\left[1/2e^{-1/2}(1-e)^{1/2}\theta^{-1/2} - 1/2e^{-1/2}(1-e)^{-1/2}\theta^{1/2}\frac{de}{d\theta} - 1/2e^{-3/2}(1-e)^{1/2}\theta^{1/2}\frac{de}{d\theta}\right]}{(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2})^2}$$

$$-K\frac{de}{d\theta}$$

$$\frac{dW}{d\theta} = \frac{e\Omega_e\left(1+e^{-1/2}(1-e)^{1/2}\theta^{-1/2}\right)}{\left(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)^2}$$

$$+\left[\frac{\Omega_e}{\left(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)} - K + \frac{\frac{\Omega_e}{2}\left(e^{1/2}(1-e)^{-1/2}\theta^{1/2} + e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)}{\left(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)^2}\right]\frac{de}{d\theta}$$

$$\frac{dW}{d\theta} = -\frac{\frac{\Omega_e}{2}\left(e^{1/2}(1-e)^{1/2}\theta^{-1/2}\right)}{\left(1+\sqrt{\theta/E}\right)^2} + \left[\frac{\Omega_e}{1+\sqrt{\theta/E}} - K + \frac{\frac{\Omega_e}{2}\left[\sqrt{\theta E}+\sqrt{\theta/E}\right]}{\left(1+\sqrt{\theta/E}\right)^2}\right]\frac{de}{d\theta}$$

(40')

Equation (41'):

$$W = \frac{e(\Omega_e,\theta,K)\Omega_e}{1+e(\Omega_e,\theta,K)^{-1/2}[1-e(\Omega_e,\theta,K)]^{1/2}\theta^{1/2}} - e(\Omega_e,\theta,K)K \qquad (38')$$

$$\frac{dW}{dK} = \frac{(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2})\Omega^e \frac{de}{dK}}{(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2})^2}$$

$$\frac{-(e\Omega_e)\left[-1/2e^{-1/2}(1-e)^{-1/2}\theta^{1/2} - 1/2e^{-3/2}(1-e)^{1/2}\theta^{1/2}\frac{de}{dK}\right]}{(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2})^2} - K\frac{de}{dK} - e$$

$$\frac{dW}{dK} = -e + \left[\frac{\Omega_e}{\left(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)} - K + \frac{\frac{\Omega_e}{2}\left(e^{1/2}(1-e)^{-1/2}\theta^{1/2} + e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)}{\left(1+e^{-1/2}(1-e)^{1/2}\theta^{1/2}\right)^2}\right]\frac{de}{dK}$$

$$\frac{dW}{dK} = -e + \left[\frac{\Omega_e}{1+\sqrt{\theta/E}} - K + \frac{\frac{\Omega_e}{2}\left[\sqrt{\theta E} + \sqrt{\theta/E}\right]}{\left(1+\sqrt{\theta/E}\right)^2}\right]\frac{de}{dK} \qquad (41')$$